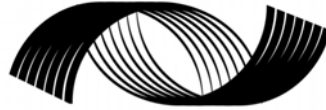




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Designing Information Markets for Decision Making

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Executive Summary

People often make important decisions based on information elicited from experts with uncertain preferences. We provide a theoretical rationale for the use of information markets in decision making tasks. Specifically, we show that markets for claims on decision-relevant variables can be efficient incentive schemes for eliciting information. Our model shows decision makers will subsidize liquidity in illiquid decision markets to gather valuable information. Our model also shows that the mere act of linking the decision to the market price will typically enhance liquidity in the market. Overall, our results highlight the potential for using information markets in diverse decision making tasks.

Designing Information Markets for Decision Making

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ABSTRACT

People often make important decisions based on information elicited from experts with uncertain decision objectives. We provide a theoretical rationale for the use of information markets in decision making tasks. Specifically, we show that markets for claims on decision-relevant variables can be efficient incentive schemes for eliciting information. Our model shows decision makers will subsidize liquidity in illiquid decision markets to gather valuable information. Our model also shows that the mere act of linking the decision to the market price will typically enhance liquidity in the market. Overall, our results highlight the potential for using information markets in diverse decision making tasks.

1 Introduction

People often make important decisions based on information gathered from individual experts. For example, suppose a farmer must decide how much seed to plant in her fields. She could pay individual experts for their rainfall predictions and use these forecasts as the basis for her seed decision. As an alternative to directly eliciting individual predictions, the

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farmer could set up a rainfall claims market and use the equilibrium price in this market as the basis for her decision.¹ In this paper, we investigate the properties of these two decision making algorithms in a stylized setting.

Our study makes three contributions to the theory of information markets and their use in decision making.² First, we provide a theoretical rationale for the use of information markets in decision making tasks.³ Second, we show that decision makers will subsidize liquidity in illiquid decision markets to gather valuable information.⁴ We also show that the decision maker can implement her desired liquidity policy either directly as the market maker or indirectly as a noise trader participating in a competitive market. Third, our model demonstrates that the mere act of linking the decision to the market price will typically enhance liquidity in the market. Thus, liquidity may be *less* of a problem in decision markets than in traditional information or asset markets.

To our knowledge, this is the first paper to provide a formal theoretical rationale for either the use of information markets in decision making or the use of liquidity subsidies in these markets. Overall, our results highlight the potential for using information markets in diverse decision making tasks.

To illustrate the structure of the model, we consider the situation of a profit-maximizing farmer, Denise, who must choose how much seed to plant in her fields. The profit-maximizing quantity of seeds depends on the uncertain quantity of rain that falls in her county this year. Two local experts, Larry and Harold, observe signals about future local rainfall. They also have different objectives for the amount of seed they would like Denise to plant. Larry would like Denise to plant fewer seeds because he is a competing farmer. Harold would like her to plant more because he is a seed producer. Both Denise and the experts care about their trading profits and production profits.

To inform her rainfall estimate and seed decision, Denise could either directly ask the experts for their information, write individual contracts with the experts to obtain their information or create an anonymous market where the experts could trade based on their

¹The term rainfall claims market refers to a market for securities that pay their owners an amount that depends on realized rainfall. The price in the rainfall claims market could be used as an input to the farmer's decision rather than as the sole basis for her decision.

²Markets for contingent claims whose value depends on information have been referred to as information markets, prediction markets or event futures markets. When the prices from these markets are used in decision making, they are usually called decision markets—*e.g.*, Hanson (2002a).

³Prior contributions that discuss decision markets include Hanson (2002a) and Ledyard (2005).

⁴Hanson (2002b) proposes liquidity subsidies for binary outcome information markets with certain desirable properties, but does not optimize these subsidies for any particular objective function.

information. In this paper, we do not model the “cheap talk” game in which Denise asks the experts, Larry and Harold, for their information, offering them no compensation. In the single-period game we consider here, this method will produce uninformative responses from the experts when their production objectives differ sufficiently from the decision maker’s (Crawford and Sobel, 1982).⁵

Instead, this paper models contracts and markets in which informed experts voluntarily participate. For example, Denise could write individual or multilateral contracts with experts offering to pay them for their information. These contracts could offer payments contingent on the realized rainfall outcome to induce experts to truthfully reveal their rainfall information—*e.g.*, see Osband (1989). Writing such contracts will not always be feasible. Denise would need to be able to identify the experts in advance and also prevent contract resale.

Alternatively, Denise could set up an anonymous market exchange for contingent contracts that pay \$1 per inch of measured rainfall next year in her county. In this case, Denise or a third-party market maker could mediate purchases and sales of rainfall contracts between experts, such as competing farmers and seed producers. This rainfall claims market would differ from conventional financial markets in that Denise would explicitly condition her seed decision on the equilibrium price. Recognizing this, interested experts would have an incentive to manipulate the price of rainfall contracts to influence Denise’s seed decision. For example, seed producers may buy rainfall contracts to increase the prevailing price and Denise’s seed purchases, even if their private information suggests future rainfall will be low. For analogous reasons, competing farmers may sell rainfall contracts to lower the prevailing price of rainfall contracts. The rainfall market resembles a multilateral contract between the farmer and experts, but does not require that Denise be able to identify trades from individual experts or prevent contract resale among experts.

Generalizing this example of the seed farmer, we derive *efficient* contracts and markets that elicit decision-relevant information. We focus on asset markets for two reasons. First, in both theory and practice, asset markets aggregate information well—*e.g.*, see Samuelson (1965) on efficient market theory, Fama (1970, 1991) on capital market evidence and Wolfers and Zitzewitz (2003) for a survey of information market evidence. Indeed, there is a growing body of literature that suggests information markets generally do better than experts in a wide variety of contexts (see, *e.g.*, Plott and Chen (2002)). Second, allowing experts

⁵Even in this case, informative reputational equilibria may exist in a repeated cheap talk game—*e.g.*, see Ottaviani and Sorensen (2005a, 2005b, 2005c) and Morgan and Stocken (2003).

to trade in asset markets imposes minimal requirements on the decision maker’s ability to distinguish and monitor experts. In the extreme, the decision maker need not be involved at all in the market process, delegating market making activity to third parties. We show that the optimal contract provides experts with incentives to reveal information similar to those provided by a market in which experts trade rainfall claims. In both our contracting and market models, some intervention by the decision maker is optimal. In particular, the decision maker is sometimes willing to sacrifice trading profits to induce experts to reveal more information.

The next section of the paper provides a brief overview of related work. Section 3 introduces the modeling framework to be used throughout the paper. Section 4 solves the optimal contracting problem for a decision maker eliciting information from experts with different production objectives from the decision maker. This section analyzes both the case in which experts’ objectives are known and the case when these objectives are unknown to the decision maker. In Section 5, we explore the properties of a simplified version of the optimal contract that does not require identifying traders. Section 6 concludes with a discussion of the applications, limitations, and directions for future research in decision markets.

2 Literature Review

Our basic modeling approach draws on insights from several areas, including contract theory, mechanism design and finance. In closely related work in contract theory, Osband (1989) examines the efficiency of contracts in motivating a forecaster to acquire and truthfully report information used in decisions. In contrast to our model, Osband (1989) assumes experts have no stake in the outcome of the decision.⁶

In the mechanism design literature, Gerardi, Postlewaite, and McLean (2005) show how a decision maker can elicit useful information from experts, but these authors do not consider contracts with contingent payments. In their model, the decision maker can approximately attain her preferred outcome when information is spread diffusely across experts.⁷ The mech-

⁶Other differences between our contracting model and Osband’s include our analysis of multiple bilateral contracts and multilateral contracts. By contrast, Osband looks at competitive bidding between multiple forecasters for the right to participate in a single bilateral contract. Our model also does not address the moral hazard problem posed by information acquisition, instead examining experts who already possess decision-relevant information.

⁷More precisely, the condition required is that experts are “informationally small” in the sense of McLean and Postlewaite (2002).

anism used by Gerardi et al. (2005) compares experts’ reports of information and punishes those experts with anomalous reports because these are less likely to be correct. Although their mechanism does not require contingent contracts, it is vulnerable to collusion among experts and ineffective when information is concentrated among a few experts. In contrast, the decision algorithms we examine are less susceptible to collusion and information concentration, but do require the use of contingent contracts.

Our treatment of decision market liquidity draws heavily on important results in finance. Milgrom and Stokey’s (1982) no-trade theorem is particularly relevant because it provides conditions under which no transactions will occur in asset markets. A lack of trading volume will severely impede the price discovery process and any attempts to use transaction prices for decision making.

Our model of a decision market builds on the microstructure model in Kyle (1985). We also adopt our definition of market liquidity from Kyle (1985), who advocates using price sensitivity to aggregate order flow. Liquidity is the inverse of price sensitivity, meaning a liquid market is one in which traders can transact in large quantities without moving prices substantially. Our model’s results on liquidity also build on Kyle (1985). Specifically, when experts have too much information relative to how much they care about influencing the decision, it will be optimal for a decision maker to provide liquidity.

Finally, our model resembles the model of price manipulation developed by Hanson and Oprea (2004), which also builds on Kyle (1985). However, our model adopts the perspective of the decision maker, whereas Hanson and Oprea (2004) do not explicitly link decisions to information market prices. By making this linkage, we can explicitly solve for the decision maker’s optimal policy conditional on market prices. In addition, we can address questions such as how should the decision maker design an information market to maximize her welfare.

3 Modeling Framework

Throughout the analysis, we assume there is an informative signal of decision-relevant information that can be verified ex post. For example, a farmer’s choice of how much seed to plant in a given year may depend on her best forecast of annual rainfall. After a year has elapsed, a verifiable signal of decision-relevant information would be the annual rainfall

measure reported by the National Weather Service for the farmer's county.^{8,9}

A decision maker will make a judgment or forecast (x) of decision-relevant information (y) based on experts' reports. For simplicity and tractability, we assume that both the decision maker and experts are risk-neutral with additively separable utility in judgment accuracy and monetary transfers. Let the decision maker's production objectives be given by $u_0(y, x) = -(y - x)^2 - t$, where y is the realization of uncertain decision-relevant information that will be revealed ex post and t is a transfer payment from the decision maker.

The interpretation is that the decision maker prefers more accurate forecasts of the decision-relevant information (y) and prefers lower transfer payments. A concrete example of decision-relevant information could be an ex post signal of rainfall for a farmer whose decision of how much seed to plant depends on rainfall. The decision maker receives greater production profits when her forecast is closer to the expected value of decision-relevant information because she can then implement a better production plan.

To maximize her utility, the decision maker can elicit information from experts. Initially, all experts and the decision maker have a prior belief about y , which we normalize to zero, so that $E(y) = 0$ and $Var(y) \equiv V_0$. If the decision maker requests advice from expert i , then this expert costlessly observes a private informative signal (s_i) that reveals further information about y , where $E(y|s_i) = s_i$ and $Var(y|s_i) \equiv V_i$. For tractability, we assume all signals s_i and the prior belief are normally distributed and independent conditional on y . After observing signal s_i , the expert updates his posterior mean of y to $E(y|s_i) = \frac{V_0 s_i}{V_0 + V_i} = v_i s_i$, where $v_i = \frac{V_i^{-1}}{V_0^{-1} + V_i^{-1}}$ and his posterior variance is $Var(y|s_i) = (V_0^{-1} + V_i^{-1})^{-1} < V_0$.

We denote expert i 's utility as $u_i(y, x) = -c(y - x + \theta_i)^2 + t_i$, where $i \in \{1, \dots, N\}$, θ_i is a parameter describing the magnitude and direction of expert i 's conflict of interest with the decision maker and t_i is the transfer payment received by expert i . The parameter $c > 0$ describes how much an expert cares about the decision outcome relative to the decision

⁸An important simplifying feature of this example is that the realization of policy-relevant information (*e.g.*, rainfall) is independent of the decision maker's policy (*e.g.*, the farmer's decision of how much seed to plant). Although we adopt this restrictive assumption in most of our models below, allowing the signal to depend on the policy choice would not change the main qualitative results.

⁹Some readers may doubt that such a verifiable signal exists in many private decision and public policy applications. For example, consider a decision maker who wants to know the *causal* impact of a tax cut on gross domestic product (GDP). Ex ante measurement of the causal effect would seem to require that an observer re-run history with and without the tax cut to measure the difference in GDP. In a related article we are writing, we show that even causal effects are amenable to the modeling framework in this article. Wolfers and Zitzewitz (2005) propose an alternative method based on instrumental variables. Abramowicz (2004) advocates the use of contracts based on ex post cost-benefit analyses of policies.

maker—*e.g.*, $c = 1$ describes an expert with equally strong production objectives, but not necessarily objectives that are well-aligned with the decision maker’s (*i.e.*, if $\theta_i \neq 0$).

In a multilateral contract with the decision maker, the expert’s transfer payment will depend on his information report (\hat{s}_i), other experts’ reports (\hat{s}_{-i}) and the realization of decision-relevant information (y). When the expert’s objective (θ_i) is unknown, the transfer payment can also depend on the expert’s reported type ($\hat{\theta}_i$). In a multilateral contract, the expert’s payment can depend on his reported signal and type and the reported signals (\hat{s}_{-i}) and types of others ($\hat{\theta}_{-i}$). For notational convenience, we denote the vector of all reported signals as $\hat{s} = (\hat{s}_i, \hat{s}_{-i})$ and the vector of all types as $\hat{\theta} = (\hat{\theta}_i, \hat{\theta}_{-i})$. We impose the budget balance constraint that the decision maker’s transfer payment must equal the sum of transfers to all experts—*i.e.*, $t = \sum_{i=1}^N t_i$.

The last part of a contract stipulates the decision rule $x(\hat{s})$ —or $x(\hat{s}, \hat{\theta})$ if experts types are unknown—that will be used by the decision maker. Ideally, the decision maker would select the optimal decision for herself given the information revealed by the expert, but this may not be feasible. In order to induce the expert to reveal decision-relevant information, the decision maker may be willing to sacrifice some autonomy. However, in this paper, we will assume that the decision maker cannot contractually commit to decision rules that are not in her best interest after observing experts’ reports.¹⁰

The decision maker’s optimal contract with an expert will, in general, depend on assumptions about the nature and magnitude of information asymmetry. In our simple setting, however, the optimal contract will not depend on the type of individual rationality constraint we impose.¹¹ This is because we focus on the class of contracts that gives all experts non-negative expected transfer payments, which is more limiting than imposing individual rationality constraints. The motivation for this restriction is that competitive offers from third parties with no control over the decision would ensure that *ex ante* transfers to experts must be weakly positive. Regardless of whether experts know their private information

¹⁰Fortunately, in all models but the model in Section 5, our assumption of no commitment has no effect on our results. Even in the last model, our qualitative results would not change if we allowed the decision maker to commit to suboptimal decision rules.

¹¹An alternative reason to relax individual rationality constraints is that participation in information elicitation mechanisms may occur for completely irrational reasons. For example, there is a voluminous literature documenting overconfidence among experts. Overconfident experts will be especially eager to participate in the mechanisms we model in this paper. Overconfident experts in our setting could easily be modeled by incorporating the realization of their private signals (s_i) in their types (θ_i).

and objectives before contracting, the non-negative transfer restriction will be binding and individual rationality constraints can be ignored.¹²

We refer to asset markets as arrangements in which contingent contracts of a specific form are traded among experts and a competitive profit-maximizing market maker (which may be the decision maker herself). The structure of these markets is analogous to the setup in Kyle (1985). In a batch auction, traders submit market orders conditional on their information and types. A market maker sets prices conditional on observed aggregate order flow. The market equilibrium results in a single clearing price,¹³ which can be used by the decision maker to improve her decision. We assume experts anticipate the dependence of the decision on the market price so they may attempt to manipulate that price.

We will think of an “asset market” as a particular kind of multilateral contract. To see the similarity for the model described above, first note that the market order of expert i takes the form $q_i = q_i(\hat{s}_i, \hat{\theta}_i)$ and the market clearing price can be described as $p(\hat{s}, \hat{\theta}) = p(\sum_{i=1}^N q_i(\hat{s}_i, \hat{\theta}_i))$. The transfer payments to experts are the experts’ profits: $t_i(\hat{s}, \hat{\theta}) = q_i(\hat{s}_i, \hat{\theta}_i)(y - p(\hat{s}, \hat{\theta}))$. The sum of the profits made by experts in the market is equal and opposite to the market maker’s (or decision maker’s) profits, which is the same budget balance constraint imposed earlier on the set of contracts. Finally, the decision maker’s decision rule can be described as $x(\hat{s}, \hat{\theta}) = x(p(\hat{s}, \hat{\theta}))$. We identify the conditions under which the types of contracts we call asset markets are optimal contracts for the decision maker. Then we examine how asset markets can be designed to meet the needs of the decision maker.¹⁴

4 Designing Optimal Contracts

In this section, we investigate what type of contract a decision maker would choose to write with experts she can uniquely identify. This analysis provides insights about the form of the optimal contract and its dependence on experts’ decision objectives and information

¹²We will prove this result in section 4.

¹³In sequential trade models such as Glosten and Milgrom (1985), there can be two prices—one for buyers and one for sellers.

¹⁴The single clearing price may seem restrictive, ruling out a mechanism similar to a limit order book. For example, it seems reasonable to allow some price discrimination based on the size of a trader’s order. But any such price discrimination would require that a market maker be able to distinguish each trader’s order. Otherwise, traders wanting to trade large orders could submit several small orders through intermediaries to obtain a better average price. The only non-manipulable anonymous market making rules are of the form $p = p(\sum q_i)$ as described above.

precision. The design of the optimal contract reflects the decision maker's desire to obtain accurate information from experts at a low cost. Specifically, the optimal contract provides transfers to experts that are equivalent to trading profits in a hypothetical market for decision-relevant information. We initially demonstrate this result for the simple case in which experts' objectives are publicly observable.

Next, we derive a similar form for the optimal contract in the case where experts' objectives are unobservable. The key parameter in these optimal contracts has an interpretation as a measure of market liquidity in the equivalent market environment. We show that the decision maker's optimal contract provides greater liquidity to experts than a competitive market maker would provide. The intuition is that greater liquidity elicits more informed trading from experts. Also consistent with this intuition, the decision maker provides the greatest liquidity subsidies to experts with greater information precision.

4.1 Contracts when Experts' Objectives Are Observable

As a useful benchmark, we first examine the simple case in which the decision maker writes a multilateral contract with N experts with known objectives (θ_i) . We assume the decision maker knows each expert i 's signal has precision V_i^{-1} , but does not know the realization of his signal (s_i) . The decision maker chooses a set of transfer payments t_i and decision rule $x(\hat{s})$ to solve:

$$\max_{x, t_i} \left\{ E_y \left[- \sum t_i(\hat{s}, y) - (y - x(\hat{s}))^2 | \hat{s}, \theta \right] \right\} \quad (1)$$

subject to incentive compatibility, individual rationality, and non-negative expected transfer constraints for the N experts.

To identify the optimal contract, it is useful to consider the decision maker's utility in the hypothetical situation where she could observe the experts' private signals directly. In this case, the decision maker would implement the posterior mean of y and make zero transfer payments to all experts. Formally, the decision maker would choose $x = E(y|s) = \frac{\sum V_i^{-1} s_i}{V_0^{-1} + \sum V_j^{-1}} \equiv x^*(s)$, where $x^*(s)$ is the optimal decision rule given the observed vector of signals. Ideally, the decision maker would also set $E(t_i) = 0$ for all i . We will denote $k_i = \frac{V_i^{-1}}{V_0^{-1} + \sum V_j^{-1}}$ so that $x^*(s) = \sum k_i s_i$. To simplify notation further, let $r_i = k_i s_i$ be an expert's true contribution to the optimal decision and \hat{r}_i be his report to the decision

maker.¹⁵ If the same decision rule $x^*(r) = \sum r_j$ and expected transfer payments $E(t_i) = 0$ are feasible when the decision maker cannot observe the experts' signals, then the contract that implements $x^*(r)$ and $E(t_i) = 0$ must be optimal.

To look for such a contract, we first assume that the decision maker will implement her posterior based on all experts' reports $x = x^*(\hat{s})$. If any expert i rejects the contract, the decision maker will implement her posterior conditional on all reported signals—*i.e.*, $x = x^*(\hat{s}_{-i})$ —to minimize the expected squared deviation of y from x . In this case, expert i receives no transfer, but still cares about the decision outcome. In equilibrium, we will verify that no expert will have an incentive to reject the transfer and decision rule offered by the decision maker. That is, the individual rationality constraint will not be violated for any expert.

Proposition 1 identifies the functional form of the transfer function that allows the decision maker to implement x^* .

Proposition 1 *When experts' objectives (θ_i) are observable, the decision maker maximizes welfare using the decision rule $x^*(\hat{r}) = \sum \hat{r}_j$ and transfer rule $t_i^*(\hat{r}, y) = 2c\theta_i(y - \sum \hat{r}_j)$.*

Proof. See Appendix A. ■

An important benefit of this particular contract is that the decision maker need not be able to commit to using the decision rule $x^*(\hat{r}) = \sum \hat{r}_j$. First, the decision maker offers the payments t^* to experts. Next, the experts will report truthfully. After learning of these reports, the decision maker will *optimally* use the rule x^* regardless of whether or not she has committed to this rule because x^* minimizes her loss function conditional on experts' truthful reports.

The optimal contract with multiple experts with known heterogeneous objectives has an interesting market analog. Consider a market in which there are contingent claims that each pay an amount y and are priced at $p(\hat{r}) = \sum \hat{r}_j$. The expert's optimal transfer function is equal to his profit from purchasing a quantity $2c\theta_i$ of these contingent contracts. The optimal transfer gives experts with more extreme types (θ_i further from zero) greater incentives to report moderate signals (\hat{r}_i closer to zero). For example, because experts with large positive objectives receive a greater number of contracts and lower signals reduce the price of these contracts, large positive objectives will have a financial incentive to understate their reports.

¹⁵Note that the posterior assumes this general form as long as the signals come from a probability distribution within the exponential family and the prior information is a conjugate prior for the signal distribution. Thus, most of the results below generalize beyond normal distributions.

This is desirable because, without this financial incentive, experts with large positive objectives would overstate their reports to influence the decision in their favor. The optimal contract balances these competing incentives perfectly, leading to truthful revelation.

Under the market interpretation, we can interpret the decision maker's decision rule as a price setting rule because $x^*(\hat{r}) = p(\hat{r}) = \sum \hat{r}_j$. The decision maker acts as a market maker that offsets any imbalance in supply and demand created by an uneven distribution of expert objectives. The total net demand of contingent contracts is $-2c \sum \theta_j$. In equilibrium, the market price and the decision depends more on the reports of experts with more precise information because $r_j = k_j s_j$ where k_j is proportional to expert j 's signal precision.

To understand the intuition behind the market-like mechanism, let's reconsider our example with the farmer trying to elicit information from rainfall experts with conflicting objectives. Suppose there is a seed producer with information about rainfall who would like the farmer to believe rainfall will be 10 inches higher than the truth, leading to increased seed planting; and there is a competing farmer with information about rainfall who would like the farmer to believe rainfall will be 10 inches lower than the truth, leading to reduced seed planting. The farmer can offer to be the counterparty to the seed producer and the competing farmer in two rainfall contract transactions, where a rainfall contract pays off \$1 per inch of measured rainfall.

Specifically, she can offer the seed producer the opportunity to purchase 10 rainfall contracts and the competing farmer the opportunity to sell 10 rainfall contracts at a price equal to the precision-weighted average of their rainfall reports. This can be thought of as asking each expert to "put his money where his mouth is." Both agents will accept these contract offers and reveal their true rainfall expectations to the farmer, anticipating that the farmer will make his seed decision based on the precision-weighted average of their rainfall reports. In this particular example, the farmer takes no net position in rainfall contracts because the seed producer and the competing farmer have exactly offsetting objectives. In the more general case, the farmer assumes a position opposing the average expert objective. He makes zero expected profit in either case.

As we will show formally in section 5, the example does not rely on the assumption that the farmer can identify the individual experts. The farmer only needs an estimate of the distribution of experts' objectives. She can then set a price and make a decision based on the aggregate of all experts' reports and her best estimate of the distribution of experts' objectives.

Interestingly, the mere act of linking the farmer's seed decision to the market price of

rainfall increases rainfall market liquidity. To see this point, we can compare the trading volume in a decision market for rainfall claims to the volume in an information market in which the farmer's seed decision is not linked to the price of rainfall. In the decision market equilibrium above, no expert is making excess trading profits and the decision maker is operating as a market maker with zero expected trading profits. Experts who want the farmer to plant more seed buy rainfall contracts at fair prices from experts who have opposite objectives. These differences in objectives are the only source of trading volume.

By contrast, in an information market equilibrium in which the seed decision is not linked to the price of rainfall, there would be no trade in the market for rainfall claims. The classic no-trade theorem of Milgrom and Stokey (1982) would apply because each of the N experts has asymmetric information about the asset's payoff. In addition, the differences in experts' decision objectives that could be used in a decision market to mitigate this information asymmetry are irrelevant in a pure information market. If the N experts submitted market orders for quantities of the contingent claims on rainfall, then no market maker would be willing to accept their orders. A decentralized market would fail, too, because no rational informed expert would be willing to conduct a bilateral trade with another rational informed expert.

4.2 Contracts when Experts' Objectives Are Unobservable

Now we extend the basic model to allow for the possibility that the decision objective (θ_i) of each expert is known only by the expert. From the perspective of the decision maker, θ_i is normally distributed with $E(\theta_i) = 0$ and $Var(\theta_i) = V_\theta$.¹⁶ For simplicity, we assume that the realizations of θ_i are independent across experts.¹⁷ We also retain the same assumptions about expert and decision maker objectives in the basic model above.

Writing an optimal contract with an expert who has private information about both his signal s_i and his type θ_i is a two-dimensional mechanism design problem. With the assumptions above, this problem can be solved by collapsing the two-dimensional uncertainty about experts into one dimension. The idea is to find a sufficient statistic for representing (s_i, θ_i) . Similar to Biais, Martimort, and Rochet (2000), the two-dimensional problem can be recast by defining the sufficient statistic as the single type.

¹⁶The problem is identical when θ_i has a non-zero expectation that is known to the decision maker.

¹⁷The form of the optimal contract does not change when experts' objectives are correlated.

To understand how this works, we examine an expert's expected utility function after receiving his signal:

$$E_y[E_{s_i, \theta_i} [t_i - c(y - x + \theta_i)^2 | s_i, \theta_i]] = -cV_i + E_y[E_{s_i, \theta_i} [t_i - c(v_i s_i + \theta_i - x)^2 | s_i, \theta_i]] \quad (2)$$

Examining the right-hand side of equation (2), we note that the expert's utility depends on his true type and signal only through the term $v_i s_i + \theta_i$, which is a sufficient statistic for (s_i, θ_i) , where $v_i = \frac{V_i^{-1}}{V_i^{-1} + V_0^{-1}}$.

Applying the sufficient statistic theorem in Holmstrom (1979), the mechanism designer can restrict herself to the class of mechanisms in which agents truthfully reveal the sufficient statistic for their types. We redefine $\eta_i = s_i + \theta_i/v_i$ as the expert's true type in the transformed problem in which the experts' incentive compatibility constraints are $\hat{\eta}_i^* = \eta_i$.¹⁸ Now the decision maker picks transfers $t_i(\eta, y)$ and a decision rule $x(\eta)$ to maximize her utility.

Once again, if the decision maker were able to directly observe η , then her problem would be much easier. She would again implement $x^* = E(y|\eta)$ and set expected transfers to zero for all experts. Note that $x^* = E(y|\eta) = \sum w_i \eta_i$, where $w_i = \frac{(V_i + \frac{1}{v_i^2} V_\theta)^{-1}}{V_0^{-1} + \sum_j (V_j + \frac{1}{v_j^2} V_\theta)^{-1}}$. If there exists a contract that replicates this outcome when η is unobservable, then this contract must be optimal.

To simplify the presentation of our results, we redefine the type and reported type once more as $\gamma_i = w_i \eta_i = w_i(s_i + \theta_i/v_i)$ and $\hat{\gamma}_i = w_i \hat{\eta}_i$. We next search for a set of transfer functions that implements the optimal decision rule $x^* = E(y|\eta) = \sum \gamma_i$ and offers expected payments of zero to all experts. As in the previous model, because this transfer function implements the decision rule $x^* = E(y|\eta)$, the decision maker does not need to commit to any decisions she will be unwilling to carry out after learning experts' signals.

Applying the same logic that led to Proposition 1, we can identify the optimal transfer rule associated with this decision rule. The proposition below describes the optimal contract:

Proposition 2 *When experts' objectives (θ_i) are unobservable, the decision maker maximizes welfare using the decision rule $x^*(\hat{\gamma}) = \sum \hat{\gamma}_j$ and transfer rule $t_i(\hat{\gamma}, y) = \alpha_{1i} \hat{\gamma}_i (y - \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j) - \bar{t}_i$, where $\alpha_{1i} > 0$, $\alpha_{2i} > 0$, and \bar{t}_i are given by:*

$$\alpha_{1i} = c(1 + \frac{v_i}{w_i}) \left(\sum_{j \neq i} w_j \right) \quad (3)$$

¹⁸Dividing the sufficient statistic by v_i is immaterial because this parameter is publicly known to all agents.

$$\alpha_{2i} = c\left(\frac{v_i}{w_i} - 1\right) \quad (4)$$

$$\bar{t}_i = cw_i \left[2V_0 \sum_{j \neq i} w_j - w_i \left(V_i + \frac{1}{v_i^2} V_\theta \right) \right] \quad (5)$$

Proof. See Appendix A. ■

The form of the optimal contract is intuitive in our farmer example. The farmer, Denise, offers transfers to the rainfall experts, Harold and Larry, that depend on their individual reports $\hat{\gamma}_i$ and the sum of their reports ($\sum_j \hat{\gamma}_j$). These transfers are designed so that both experts have an incentive to truthfully reveal their types $\gamma_i = w_i(s_i + \theta_i/v_i)$, which depend on their rainfall signals (s_i) and seed planting objectives (θ_i). All else equal, the seed producer, Harold, will announce a higher rainfall report than the competing farmer, Larry, because Harold prefers more seed to be planted than Larry (*e.g.*, $\theta_H > \theta_L$). However, both experts will incorporate their rainfall signals in their rainfall reports, too.

In making her seed planting decision, Denise chooses a seed level equal to the sum of Harold and Larry's reports ($x^*(\hat{\gamma}) = \sum \hat{\gamma}_j$). Although she cannot be assured that Harold and Larry have exactly offsetting objectives, by aggregating their reports Denise will reduce the influence of idiosyncratic variation in objectives on her decision. In addition, because all experts's reports contain their signals about a common rainfall amount, the information contained in their reports will be preserved in the aggregation process. Thus, Denise's decision will reflect as much information-based reporting and as little manipulative reporting as possible.

Once again, the optimal transfer function has an interesting market analog. It is the profit function for expert i who buys $\hat{\gamma}_i$ contingent claims that each pay an amount y and are priced at $p_i(\hat{\gamma}) = \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j$. The parameter α_{1i} is a multiplicative weight that amplifies the expert's profit in the market. In equilibrium, the decision maker sets this parameter just high enough to offset the expert's desire to influence the decision in his favor. That is, α_{1i} ensures that the expert's profit motivation counteracts his policy objectives. The parameter α_{1i} can be viewed as an exchange rate between decision utility and money. Intuitively, when an expert cares more about the decision (higher c), then his profits in the market must be greater to motivate him to reveal his information rather than his decision objectives.

The pricing function $p_i(\hat{\gamma}) = \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j$ adopted by the decision maker also has an interesting interpretation. Because the decision maker can distinguish among experts with different signal quality (V_i^{-1}), he chooses a pricing function that discriminates to maximize

his surplus. The ratio $\frac{\alpha_{2i}}{\alpha_{1i}}$ describes the sensitivity of price to an expert's quantity. The inverse of price sensitivity can be interpreted as market liquidity.

There is a single market clearing price when all experts have the same signal quality ($V_i^{-1} = V$) because $\alpha_{1i} = \alpha_1$ and $\alpha_{2i} = \alpha_2$, implying $t_i(\hat{\gamma}, y) = \alpha_1 \hat{\gamma}_i (y - \frac{\alpha_2}{\alpha_1} \sum_j \hat{\gamma}_j) - \bar{t}$. In this case, the market depicted here is very similar to the competitive market modeled in Kyle (1985). The main differences are the entrance fee \bar{t} and the particular liquidity parameter chosen by market maker, who is also the decision maker here. The competitive market maker in Kyle (1985) does not make any decision based on the market price, whereas the market maker here is also selecting $x = \sum \hat{\gamma}_i$. Rather than setting the price equal to the optimal decision as in the previous example, the decision maker may find it optimal to set $p_i(\hat{\gamma}) = \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j = \frac{\alpha_{2i}}{\alpha_{1i}} x$, where $\frac{\alpha_{2i}}{\alpha_{1i}} \neq 1$. We explain the intuition for this result below.

The liquidity parameter $\frac{\alpha_2}{\alpha_1}$ is directly analogous to Kyle's λ parameter. If this market were competitive, then the market maker would set prices according to the conditional expectation of y depending on order flow $\sum \hat{\gamma}_i$. From above, we know this would be the pricing rule of $p_i(\hat{\gamma}) = \sum_j \hat{\gamma}_j$, which implies that $\lambda = \frac{\alpha_{2i}}{\alpha_{1i}} = 1$. For the decision market presented here, we can show the following:

Proposition 3 *When V_θ is sufficiently small, then $\frac{\alpha_2}{\alpha_1} < 1$.*

Proof. See Appendix A. ■

Recall that $\frac{\alpha_2}{\alpha_1}$ is a measure of illiquidity or the inverse of liquidity. In words, the proposition states that the decision maker will provide greater liquidity than a competitive market maker ($\frac{\alpha_2}{\alpha_1} < 1$) when there is sufficient asymmetric information—*i.e.*, there are few experts trading based on their decision objectives relative to those trading based on their information. A decision maker who provides greater liquidity than a zero-profit market maker will expect to incur losses from her market making activity. The idea is that the decision maker subsidizes liquidity in the decision market to elicit valuable information from experts via their trades. Such a subsidy makes sense only when there is a lot of valuable information (large V_0) relative to the amount of trading based on uncertainty in experts' decision objectives (small V_θ). The benefits of the subsidy depend on the information revealed in the price. The cost of providing liquidity is offering better deals to traders trying to influence the decision—*i.e.*, lower prices to buyers and higher prices to sellers.

Another interpretation of Proposition 3 is that liquidity provided by manipulative trade and liquidity provided by the decision maker are substitutes. When there is sufficient manipulative trade to motivate informed trade, then the decision maker does not need to subsidize

liquidity. Supporting this interpretation, we have also shown in results not presented here that allowing correlation between experts objectives decreases the liquidity provided by the decision maker. When experts objectives are correlated, their manipulative trades will not cancel each other in the aggregate. This means that total order flow will depend more on the realizations of experts' objectives (θ), encouraging increased information-based trade and diminishing the need for the decision maker to subsidize liquidity. The results in section 5 further reinforce this interpretation.

Proposition 3 does not specify exactly how small V_θ needs to be relative to V_0 for the decision maker to subsidize liquidity. To gain some intuition, consider the simple case in which there are many experts ($N \rightarrow \infty$) who each observe perfect signals of y ($V_i = 0$). In this situation, the decision maker will subsidize liquidity if and only if $V_\theta < V_0$. If the variance in experts objectives is less than the variance in their informative signals, then the decision maker can improve her welfare by subsidizing liquidity to obtain better information from prices.

In the optimal contract, the decision maker takes back this expected liquidity subsidy from the experts opting to trade in the market by levying the entrance fee \bar{t} . One can think of \bar{t} as the expected loss of the market maker who offers a liquidity schedule with price sensitivity $\frac{\alpha_2}{\alpha_1}$. As shown above in the general case in which experts signal precision may differ, the optimal pricing schedule supplied to an expert $\frac{\alpha_{2i}}{\alpha_{1i}}$ will depend on the expert's signal precision. The following proposition identifies the experts to whom the decision maker would offer the greatest liquidity subsidies:

Proposition 4 *When V_θ is sufficiently small, then $\frac{\partial \frac{\alpha_{2i}}{\alpha_{1i}}}{\partial V_i} > 0$.*

Proof. See Appendix A. ■

In situations where the decision maker would like to subsidize liquidity (*i.e.*, small V_θ), the decision maker is willing to subsidize liquidity ($\frac{\alpha_{1i}}{\alpha_{2i}}$) most for the experts with better information (higher V_i^{-1}). Again, the intuition is that subsidizing liquidity motivates more informed trading, which improves the accuracy of the decision because it is based on the market price. If it is possible to identify which traders have better information, then targeting the liquidity subsidy toward these traders is efficient.

4.3 Discussion

The results above should not be taken too literally because our model's assumptions do not capture many of the complexities of empirical environments. First, real experts may not be

risk-neutral and do not have unlimited wealth to wager in markets. The contract described above requires that experts with extreme type values (γ_i) purchase extremely large trading positions, which may be infeasible for wealth-constrained experts or unacceptable for risk-averse experts. Second, it is not costless to participate in a market and acquire information. Adding costly information acquisition to our model is likely to increase the decision maker’s liquidity subsidy through the channel described in Hanson and Oprea (2004). In addition to motivating increases in the strength of bets given their information, a liquidity subsidy would motivate increased information collection by traders.

Our model does not capture certain features of markets that encourage trade even without liquidity subsidies. For example, there is mounting evidence that experts and traders are overconfident in the precision of their private information. If this is true, then many agents will trade in the market even without liquidity subsidies or differences in decision objectives. Moreover, the individual rationality constraints may be relaxed somewhat because each expert has an inflated expectation of his expected trading profits in the market.

Notwithstanding its limitations, the model here demonstrates that a properly designed market may be a practical means for collecting decision-relevant information from experts with uncertain decision objectives. Our results also suggest using a specific market intervention: a liquidity subsidy to facilitate information collection. The desired extent of this intervention will depend on the precision of experts’ information and the uncertainty in their decision objectives.

5 Linking Contracts to Markets

To show an exact equivalence between the optimal contract and a decision market, we have sacrificed some realism. In particular, the decision market portrayed above requires that the decision maker be able to distinguish among traders, which is not possible in many situations. Moreover, there are alternative modeling assumptions in which the correspondence between the optimal contract and a decision market will not be exact. The point of the previous model is not to suggest that decision markets of the form described above are always optimal, but to demonstrate why decision markets should be considered viable decision making tools.

In this section, we make two realistic modifications to the optimal contract above that naturally lead us to consider a simpler decision market, which shares many of the features of Kyle’s (1985) market microstructure model. First, we suppose that decision makers cannot identify individual traders. Second, we suppose that the decision maker cannot commit to a

decision rule that is not in her best interest after the market clears. Appendix B provides a more formal discussion and motivation for these assumptions.

We then show that our most important qualitative insights into contract design (*e.g.*, Propositions 2 and 3) also apply to the design of this anonymous decision market. The decision maker again makes a decision based on the market clearing price, which is a linear function of traders' signals and their decision objectives. Again, to improve the quality of information on which her decision is based, the decision maker subsidizes liquidity in illiquid decision markets with little manipulative trade.

This modeling approach has three main benefits. First, it is reassuring that our results generalize to more traditional market models, such as Kyle (1985). Second, the tractability of the Kyle-type model facilitates comparisons between different market designs, such as the equilibrium under a decision market maker versus a competitive market maker. Third, the familiarity of the Kyle-type framework is helpful for providing intuition into optimal market-based decision rules and liquidity subsidies.

5.1 Modeling Decision Markets

In our model of an anonymous decision market, we make several simplifying assumptions to obtain tractable solutions. Again, we consider a single decision market in which the information desired is unrelated to the alternative chosen—*e.g.*, the farmer's choice of how much seed to plant does not affect rainfall. There is only one uncertain piece of information y as in the original model.

Our model of a decision market closely resembles Kyle (1985). We consider a standard market for contingent contracts, each yielding an uncertain amount y , but we add the feature that the decision maker selects a decision x after observing the market clearing price for contingent contracts. We assume there is a single informed trader who observes a perfect signal of decision-relevant information, implying that $s = y$. This informed trader has no interest in the decision selected (x). However, there is also a single manipulator with quadratic decision objectives described by θ as in the earlier models. The manipulator has no private information about y .

Traders submit market orders to a market maker who sets a clearing price in a batch auction. We will consider the equilibrium under both a competitive market maker (as in Kyle (1985)) and a decision maker who sets an asset price p in response to aggregate order

flow Q from traders. The decision maker can only make the market if he offers traders a better price than a competitive market maker.

The informed trader submits his market order q_I before he knows the equilibrium price, but after viewing his private signal s . Total order flow Q comes from the two anonymous traders: the informed rational trader and the uninformed manipulator. Thus, $Q = q_I + q_M$, where q_I is the quantity demanded by the informed trader and q_M is the quantity demanded by the manipulator. We consider only a single informed trader and a single manipulator because our model's qualitative results depend only on the aggregate order flow from informed traders and manipulators, not on the composition of these groups.¹⁹

The market maker aggregates the informed and manipulator orders and forms a pricing rule $p(Q)$. Note that $p = E(y|Q)$ is the competitive rule, which guarantees zero profits. In theory, the decision market maker can select any pricing rule in which she incurs expected losses for any order flow amount. Following Kyle (1985), we consider only linear pricing rules. This restriction simplifies the exposition and preserves the analogy to the optimal contract examined in section 4.

We also assume the decision maker cannot commit to a suboptimal decision after observing the price, implying that her decision rule is $x = E(y|Q)$. Given his signal s , the informed trader chooses his asset demand to maximize his expected trading profits, which implies:

$$q_I \in \arg \max_{\hat{q}_I} \{E_y [E_\theta [\hat{q}_I(y - p(Q(\hat{q}_I)))]|s]\} \quad (6)$$

We look for an equilibrium in which the market maker's price depends linearly on order flow, traders orders depend linearly on their private signals and decision objectives, and the decision maker's policy rule depends linearly on the asset price:^{20,21}

$$p = \lambda Q \quad (7)$$

$$q_I = b_I s \quad (8)$$

$$q_M = b_M \theta \quad (9)$$

¹⁹A formal proof of this point is available from the authors upon request.

²⁰There is also a symmetric linear equilibrium in which there are many identical informed traders and manipulators. Symmetric refers to the fact that all traders of the same type use the same quantity strategy, but do not necessarily submit the same actual quantities.

²¹In this linear equilibrium, the constants in the pricing rule, quantity strategies, and decision rule must be zero as a result of our assumption that the common prior estimate of y is zero.

$$x = kp \tag{10}$$

We assume that the manipulator's decision objective θ is normally distributed as $N(0, V_\theta)$. We also assume y is normally distributed as $N(0, V_0)$.

We must verify that such an equilibrium exists. For the case in which the market maker is competitive, Proposition 5 characterizes the market maker's equilibrium pricing rule, decision rule, and their comparative statics along with traders' equilibrium strategies.

Proposition 5 *A competitive market maker for decision-relevant information will set $\lambda_c = \frac{\sqrt{V_0}}{c(2\sqrt{V_\theta} - \sqrt{V_0})}$ when $V_\theta > \frac{V_0}{4}$ and refuse to operate otherwise. The decision rule is $x = p = s/2 + \theta/2$ when the market maker is competitive. A competitive market maker supplies more liquidity (λ_c^{-1} is higher) when the manipulator has stronger objectives (c increases) or more uncertain objectives (V_θ increases), or the informed trader has less information (V_0 decreases). The informed trader will follow the strategy $q_I = \frac{c(2\sqrt{V_\theta} - \sqrt{V_0})}{2\sqrt{V_0}}s$ and the manipulator will follow the trading strategy $q_M = \frac{c(2\sqrt{V_\theta} - \sqrt{V_0})}{2\sqrt{V_\theta}}\theta$ when $V_\theta > \frac{V_0}{4}$. Otherwise, both traders will not trade.*

Proof. See Appendix A. ■

A few remarks are in order. First, a competitive market maker will not supply any liquidity unless $V_\theta > \frac{V_0}{4}$. If the asymmetric information problem is too severe, then the competitive market maker cannot offer any liquidity and still make zero profits. Second, the competitive market maker supplies more liquidity when the asymmetric information problem is less severe ($\frac{d\lambda_c}{dc} < 0$, $\frac{d\lambda_c}{dV_\theta} < 0$ and $\frac{d\lambda_c}{dV_0} > 0$). This result is analogous to Kyle's (1985) liquidity results.

Third, because the competitive market maker always sets a fair price such that $p = E(y|Q)$, the decision maker can directly use this unbiased estimator of decision-relevant information in her decision rule—i.e., $x = p$. Intuitively, $k = 1$ because the market price is the minimum variance, unbiased estimate of decision-relevant information.²² The decision maker wants to implement her best estimate of decision-relevant information and she can do no better than the competitive market maker who observes order flow information.

Fourth, the decision maker sets $x = p = \lambda_c Q = s/2 + \theta/2$. Exactly analogous to the contract model in section 4, the decision maker selects a judgment based on a linear combination of traders' signals and their objectives. In this special case, signals and objectives are given equal weights in the decision maker's judgment.

²²The market price has the minimum variance among all unbiased *publicly* available estimators of decision-relevant information. By assumption, the informed trader has perfect private information ($s = y$).

Now consider the case in which the decision maker also makes the market for decision-relevant information. The decision maker will anticipate using a decision rule $k(\lambda)$ when she picks her liquidity parameter λ to maximize her utility. Her utility depends on both the conditional variance of decision-relevant information and her liquidity subsidy costs. The decision maker will incur a trading loss if she subsidizes liquidity provision—*i.e.*, sets λ below λ_c —but she may gain by increasing the informativeness of order flow. Recall that the decision maker cannot earn positive profits as a market maker because she faces perfect competition from other uninformed market makers.

Before explicitly solving for the equilibrium in this market, we note two interesting features of the equilibrium. First, there is a clear distinction between a decision market and an information market. When $k = 0$, the market described above is an information market because there is no decision linked to the market price. No uninformed trade would occur in this information market because there is no reason to manipulate the price. Thus, prices would be perfectly revealing, negating the incentive for the informed trader to trade. However, when $k > 0$ and the price in the market is linked to the decision, the manipulator will want to influence the price to affect the decision. This enables the informed trader to profit, so trade will take place.

Second, when there is no uncertainty in whether the manipulator will want to influence the decision (*i.e.*, $c = 0$ or $V_\theta = 0$), a competitive market maker will refuse to operate. In this case, the insider is unwilling to trade because prices would be perfectly revealing if he did trade. However, a decision market maker would be willing to operate at a loss up to V_0 , which is the (monetized) value of the insider's perfect information.²³ Pushing this logic further, we can ask whether the decision market maker will be willing to operate in other situations in which adverse selection is so great that the competitive market maker would not be willing to operate—*e.g.*, $0 < V_\theta \leq \frac{V_0}{4}$. Proposition 6 addresses this question by characterizing the equilibrium when the decision maker makes the market.

Proposition 6 *When $V_\theta \geq \frac{V_0}{4}(1+c^{-1})^2$, a decision maker who makes a market for decision-relevant information will select a pricing rule $p = \lambda^*Q$ such that $\lambda^* = \lambda_c$, where λ_c is the corresponding pricing rule for a competitive market maker satisfying $\lambda_c = \frac{\sqrt{V_0}}{c(2\sqrt{V_\theta}-\sqrt{V_0})}$. At these values of V_θ , the decision maker will not intervene in the market. However, for intermediate values in which $\frac{V_0}{4} < V_\theta < \frac{V_0}{4}(1+c^{-1})^2$, the decision maker will act as a market maker with a price sensitivity of $\lambda^* < \lambda_c$. Furthermore, for $0 < V_\theta \leq \frac{V_0}{4}$, the decision maker*

²³This can be seen by computing the difference between the decision maker's quadratic decision outcome utility under perfect information and no information.

will set a finite λ^* , whereas a competitive market maker will not operate. In the cases in which $0 < V_\theta < \frac{V_0}{4}(1 + c^{-1})^2$, the decision market maker incurs non-zero trading losses, but obtains more precise decision-relevant information than the competitive market maker.

Proof. See Appendix A. ■

Proposition 6 makes two important and related points. First, the decision maker never provides less liquidity than a competitive market maker. In particular, the decision maker provides strictly more liquidity in illiquid markets where there is little manipulative trade. This result is analogous to Proposition 3, which describes the liquidity subsidy provided by a decision maker in the optimal contract with uniquely identifiable traders.

Second, the decision market maker will always operate in the situations where a competitive market maker would refuse to operate—*i.e.*, when $0 < V_\theta \leq \frac{V_0}{4}$. In these situations, there is insufficient manipulative trade to overcome the adverse selection problem faced by a competitive market maker. In other words, a competitive market maker will be unable to make zero profits by providing *any* liquidity, so she will refuse to operate. However, for a decision market maker, information-based trade has the offsetting benefit that it improves her decision. Thus, she is willing to provide some liquidity despite the severe adverse selection problem that she faces when she trades with informed traders.

In illiquid markets where uncertainty in the manipulator's objectives is low ($0 < V_\theta < \frac{V_0}{4}(1 + c^{-1})^2$), Proposition 6 states that the decision market maker sets price sensitivity below the competitive level ($\lambda^* < \lambda_c$), which implies that the decision market maker must be subsidizing liquidity. Lowering price sensitivity below the competitive level will always lead to trading losses because profits increase with greater price sensitivity and the competitive level of profits is zero. In terms of her utility, the trading losses incurred by the decision maker are outweighed by the decision gains she reaps from increased information-based trade.

Although the model in this section has maintained the assumption that the decision maker behaves as a market maker, none of the results would change if we instead treat the decision maker as a rational uninformed trader. As an uninformed trader, the decision maker would also want to subsidize liquidity. She could do this by submitting a random order to a competitive market maker, who would endogenously provide greater liquidity in response to this random order flow.

To prove this point, consider a model in which the decision maker submits a random order ρ , where $\rho \sim N(0, V_\rho)$, to a competitive market maker. The decision maker selects the distribution of the order, but cannot control the specific realization of her order. Because the single parameter V_ρ is a sufficient statistic for the distribution of ρ , we can think of the

decision maker as choosing V_ρ . For simplicity, we suppose that only the decision maker can observe the realization of ρ , which occurs after the order is submitted but before the market clears. This will allow the decision maker to “debias” the market price by accounting for the effect of the realization of ρ . The competitive market maker only observes aggregate order flow, but is aware that the decision maker’s random order flow is included in aggregate order flow.²⁴ In this random trading model, we prove the following result:

Proposition 7 *The equilibrium market liquidity (λ^{-1}), decision rule, and manipulator and informed trading strategies are the same whether a rational decision maker optimally trades in the market or optimally makes the market.*

Proof. See Appendix A. ■

Proposition 7 shows that the expected equilibrium in the random trading model is the same as the equilibrium in the market making model. The main point is that our conclusion about the decision maker’s participation in the market is not dependent on our assumption that the decision maker can act as a market maker. She can achieve the *same* expected utility and outcome by randomly trading in the market with a competitive market maker. Proposition 7 raises the interesting question of whether agents who behave as noise traders are merely doing so to learn more from the market price. Before dismissing such agents as irrational, future theoretical and empirical work should investigate this possibility.

To further investigate the comparative statics and gain intuition, we numerically solve for the price sensitivity value set by a decision market maker (λ^*). In all numerical solutions, we hold the informed trader’s private information constant ($V_0 = 1$) and allow uncertainty in the manipulator’s objectives (V_θ) to vary because our results depend only on the ratio of uncertainty in the manipulator objectives to private information of the informed trader (V_θ/V_0). We also allow the strength of manipulators’ objectives (c) to vary. We supplement these numerical solutions with our analytical solution for the competitive market maker’s price sensitivity (λ_c).

Figure 1 depicts a series of solutions for the price sensitivities of the decision market maker (λ^*) and the competitive market maker (λ_c). The liquidity parameters set by the decision

²⁴In practice, writing a contract to enforce this random order would not be too difficult. For example, suppose a decision maker submits an order to buy ρ contracts, where ρ is the realization from a third-party random-number generator. The distribution of ρ would need to be public, but the realization could be public (*e.g.*, the point difference in the SuperBowl) or private (*e.g.*, the spin on a roulette wheel with normally distributed values). The decision maker’s random order would need to be a binding contract with the market maker that would be verified *ex ante* and enforced *ex post* by a neutral third party. Anticipating the distribution of this random order will be a part of total order flow, a competitive market maker would lower the sensitivity of price to order flow.

maker and competitive market maker vary with both the strength of the manipulator's decision objectives (c) and the uncertainty in his objectives (V_θ).²⁵ Figure 1 shows plots of λ^* and λ_c versus V_θ with the value of c set at 1.0—*i.e.*, the manipulator's objectives are just as strong as the decision maker's. Thus, each plot depicts the relationship between the decision market maker's price sensitivity (λ) and uncertainty in the manipulator's objectives (V_θ).

[Insert Figure 1 around here.]

In both plots in Figure 1, price sensitivity declines with uncertainty in the manipulator's decision objectives (V_θ). In fact, when uncertainty in the manipulator's objectives approaches zero, both the decision maker and the competitive market maker supply no liquidity at all.²⁶ However, the competitive market maker shuts down when uncertainty in the manipulator's objectives is small ($V_\theta < \frac{V_0}{4}$), whereas the decision market maker still accepts some orders when there is little uncertainty in the manipulator's objectives ($0 < V_\theta < \frac{V_0}{4}$). The numerical results also show that a decision market maker will sometimes subsidize liquidity even when a competitive market maker would be willing to operate—*e.g.*, $V_\theta > \frac{V_0}{4}$.

Figure 1 confirms the relationship established in Proposition 6: the decision maker's price sensitivity (λ^*) is lower than a competitive market maker's price sensitivity (λ_c) when uncertainty in the manipulator's objectives (V_θ) is small. The interpretation is that the decision maker subsidizes liquidity when the market is most illiquid. Furthermore, the figure confirms that the decision market maker always supplies at least as much liquidity as the competitive market maker—*i.e.*, λ^* is never higher than λ_c . This result follows almost directly from our assumption of competition in market making. Intuitively, raising price sensitivity (λ) above the competitive level would imply greater than zero profits, which is not sustainable because uninformed market makers would drive these profits to zero.

The figure reveals that a decision market maker will set the same liquidity parameter as a competitive market maker when either the manipulator's decision objectives are sufficiently uncertain (large V_θ). In other words, when the manipulator is already supplying a lot of liquidity to the market, the decision maker will not subsidize liquidity. In these cases, liquidity subsidies are not needed because the price manipulator is already attracting order flow from the insider who is trading intensely based on his information. A simple interpretation of

²⁵We also obtain solutions for k , which range between 1/2 and 1. $k < 1$ when the decision maker is subsidizing liquidity and $k = 1$ in an unsubsidized competitive market, regardless of the market maker.

²⁶In numerical results not shown here, we have verified that almost all of the comparative statics that apply to the uncertainty in the manipulator's objectives (V_θ) also apply to the strength of the manipulator's objectives (c). This is consistent with Propositions 5 and 6.

this result is that the liquidity supplied by the manipulator is a substitute for the liquidity subsidy provided by the decision maker.

[Insert Figure 2 around here.]

Figure 2 illustrates the trading profits of the market maker, the informed trader and the manipulator for different levels of variance in the manipulator’s objectives. When the variance in the manipulator’s objectives is low, he undertakes small positions. This small uninformed order flow leads to low market liquidity, making the informed trader reluctant to place large orders. In this case, the market maker must subsidize liquidity in order to elicit information from the informed trader. Thus, Figure 2 confirms the idea that the liquidity subsidy is sometimes a substitute for uninformed order flow.²⁷

6 Conclusion

This paper builds on theories in mechanism design, contracts and finance. Unlike previous analyses, we focus on how to write contingent contracts with experts who have a vested interest in the decision. This means that the experts will consider how their forecasts or market behavior will influence the decision.

We derive three key results. First, we provide a theoretical rationale for the use of information markets in decision making tasks. Specifically, we show that markets for claims on decision-relevant variables can be efficient incentive schemes for eliciting information. Although these markets are somewhat unconventional in that the decision maker offers each trader a different price schedule and charges an entrance fee, the contingent transfer payments between agents are equivalent to trades in decision markets. If a decision maker cannot identify different traders, then she cannot offer different price schedules and levy entrance fees, making this market more conventional.

Second, our model shows that a decision maker will subsidize liquidity in illiquid decision markets to gather valuable information. In our Kyle-type model, the decision maker always provides at least as much liquidity as a competitive market maker and often provides more. There are even some situations in which a competitive market maker would refuse to operate, but a decision maker would be willing to supply some liquidity and incur expected trading losses. From the decision maker’s perspective, some trade in a market for decision-relevant

²⁷At extremely low values of variance in the manipulator’s preferences, the liquidity subsidy and liquidity supplied by manipulators are complements. We do not emphasize this effect because it arises from the well-known adverse selection problem faced by all market makers and is not unique to the model here.

information is better than none. We also show that the decision maker can implement her desired liquidity policy either directly as the market maker or indirectly as a noise trader participating in a competitive market.

Third, our model demonstrates that the mere act of linking the decision to the market price will typically enhance liquidity in the market. Thus, liquidity may be less of a problem in decision markets than in traditional information or asset markets. In decision markets, experts may choose to trade because they have strong decision objectives. Moreover, this manipulative trading is a substitute for the liquidity subsidy provided by a decision maker. This can be compared with the more conventional information market case in which even experts with decision objectives will not trade. In both markets, however, liquidity subsidies may be useful for obtaining information. Although we derive special cases of optimal liquidity subsidies in this paper, the form of the optimal subsidies in more general settings requires further investigation.

We believe that information and decision markets are likely to become more prevalent in the future.²⁸ While we have developed a parsimonious theory of how such markets could be designed efficiently, it remains to be seen whether such mechanisms will actually work well in practice.

More research is needed to understand the properties of different kinds of decision markets. For example, little is known about the theoretical properties of decision markets in which the decision itself has an impact on the realization of decision-relevant information. Applied research, including both laboratory and real-world experiments, can assess how robust different decision markets are in practice. Whereas information markets have been found to forecast extremely well, decision markets have not been studied empirically. There are compelling theoretical reasons to expect different behavior in decision markets, but theory alone cannot address this issue.

²⁸It is interesting to note, however, that decision makers informally condition their policies on market prices as a matter of routine. For example, firms condition their decisions of whether to launch a new product on prevailing prices in related markets. Firms often decide whether to issue equity based on recent equity returns. The Federal Reserve decides whether to cut interest rates based on inflation rates.

Appendix A

In this appendix, we prove Propositions 1, 2, 3, 4, 5, 6 and 7.

Proposition 1: When experts' decision objectives (θ_i) are observable, the decision maker maximizes welfare using the decision rule $x^*(\hat{r}) = \sum \hat{r}_j$ and transfer rule $t_i^*(\hat{r}, y) = 2c\theta_i(y - \sum \hat{r}_j)$.

Proof: Exploiting the revelation principle (*e.g.*, Myerson, 1979), we can restrict our search for optimal contracts to those in which all experts truthfully report their private signals (r_i). Calling expert i 's report of his update to his posterior \hat{r}_i , expert i 's incentive compatibility constraint can be written:

$$r_i \in \arg \max_{\hat{r}_i} \{E_y[E_{r_{-i}} [t_i(\hat{r}_i, \hat{r}_{-i}, y) - c(y - x(\hat{r}_i, \hat{r}_{-i}) + \theta_i)^2 | r_i]]\} \quad (11)$$

The individual rationality constraint is:

$$E_y[E_{r_{-i}} [t_i(r, y) - c(y - x(r) + \theta_i)^2 | r_i]] \geq -c(V_0^{-1} + \sum_{j \neq i} V_j^{-1})^{-1} - c\theta_i^2 \quad (12)$$

under the assumption that the decision maker will implement his best guess based on all $N-1$ participating experts' signals if one expert unilaterally opts out of the mechanism. The individual rationality constraint (12) will be satisfied by the proposed mechanism because all experts will receive expected utility $-c(V_0^{-1} + \sum_j V_j^{-1})^{-1} - c\theta_i^2$ from participating, which is greater than $-c(V_0^{-1} + \sum_{j \neq i} V_j^{-1})^{-1} - c\theta_i^2$. Intuitively, expected transfers and decision influence are zero regardless of participation, so it makes sense to participate and improve the quality of information used in decision making.²⁹

To satisfy the incentive compatibility constraint (11), it is sufficient for the expert utility function in 11 to satisfy the first- and second-order conditions for the maximization in (11). So the decision maker must choose transfers $t_i(\hat{r}, y)$ to satisfy these equations. After substituting the conjectured decision rule $x^*(\hat{r}) = \sum \hat{r}_j$, the simplified first-order condition for incentive compatibility is:

$$E_y \left[E_{r_{-i}} \left[\frac{\partial t_i(r, y)}{\partial \hat{r}_i} \right] \right] = -2c\theta_i \quad (13)$$

²⁹Because the optimal contract must satisfy incentive compatibility, each expert will report his information truthfully if he participates. Thus, the expert's decision preferences will not affect his information report and will not influence the decision.

When an expert has decision objectives that are aligned with the decision maker's ($\theta_i = 0$), no incentive is necessary to motivate this expert to reveal his information. He is intrinsically motivated to reveal information by his desire to reduce his own loss function from the decision outcome.

More generally, we can integrate both sides of this equation with respect to \hat{r}_i to gain insight into the form of the optimal transfer function. The result of this integration must contain a term of the form $-2c\theta_i\hat{r}_i$. Furthermore, note that the second-order incentive compatibility condition is satisfied because the expert's loss from the optimal decision rule is a negative quadratic function of (*i.e.*, concave in) \hat{r}_i .

In addition, the decision maker would like to reduce each expert's expected transfers to zero to maximize her utility. This can be done without altering incentive compatibility by combining the $-2c\theta_i\hat{r}_i$ with another term that does not depend on expert i 's report and makes the expected transfer zero. An intuitive guess for the optimal contract is a transfer of the form:

$$t_i^*(\hat{r}, y) = 2c\theta_i(y - \sum \hat{r}_j) \quad (14)$$

It is straightforward to verify that this transfer has an expectation of zero and satisfies the first- and second-order conditions for incentive compatibility. Therefore the decision rule x^* and the transfer payments t^* constitute an optimal contract for the decision maker. QED.

Proposition 2: When experts' decision objectives (θ_i) are unobservable, the decision maker maximizes welfare using the decision rule $x^*(\hat{\gamma}) = \sum \hat{\gamma}_j$ and transfer rule $t_i(\hat{\gamma}, y) = \alpha_{1i}\hat{\gamma}_i(y - \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j) - \bar{t}_i$, where $\alpha_{1i} > 0$, $\alpha_{2i} > 0$, and \bar{t}_i are given by:

$$\alpha_{1i} = c(1 + \frac{v_i}{w_i}) \left(\sum_{j \neq i} w_j \right) \quad (15)$$

$$\alpha_{2i} = c(\frac{v_i}{w_i} - 1) \quad (16)$$

$$\bar{t}_i = cw_i \left[2V_0 \sum_{j \neq i} w_j - w_i \left(V_i + \frac{1}{v_i^2} V_\theta \right) \right] \quad (17)$$

Proof: The first step is to identify all incentive compatible transfer functions when the optimal decision rule is used. Substituting the transfer $t_i(\hat{\gamma})$ and decision rule $x = \sum \hat{\gamma}_i$ into expert i 's utility function (2) gives:

$$-cV_i + E_y \left[E_{s_{-i}, \theta_{-i}} \left[t_i(\hat{\gamma}_i, \hat{\gamma}_{-i}) - c(v_i s_i + \theta_i - \sum \hat{\gamma}_j)^2 | s_i, \theta_i \right] \right] \quad (18)$$

After differentiating (18) and imposing incentive compatibility $\hat{\gamma}_i = \gamma_i$ in the first-order condition, we obtain:

$$E\left(\frac{\partial t_i}{\partial \hat{\gamma}_i}\right) = -2c \left[v_i s_i + \theta_i - \gamma_i - E\left(\sum_{j \neq i} \gamma_j\right) \right] \quad (19)$$

After further simplification, we can express the expected marginal transfer in terms of the expert's privately known signal (s_i) and decision objective (θ_i):

$$E\left(\frac{\partial t_i}{\partial \hat{\gamma}_i}(\hat{\gamma}_i = \gamma_i)\right) = -2c \left[\left(v_i \left(1 - \sum_{j \neq i} w_j \right) - w_i \right) s_i + \left(1 - \frac{w_i}{v_i} \right) \theta_i \right] \quad (20)$$

The linearity in signals and types suggests a strategy for choosing an optimal transfer function. We will consider a transfer function which is quadratic in $\hat{\gamma}_i$, substitute this function in the equation above and match the coefficients on s_i and θ_i in the equation above. Our transfer function must have at least two free parameters to ensure that we can match the two coefficients.

Specifically, we suppose that $t_i(\hat{\gamma}, y) = \alpha_{1i} \hat{\gamma}_i (y - \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j) - \bar{t}_i$ is an optimal transfer function, where \bar{t}_i is a lump sum payment to expert i . To examine this conjecture, we differentiate this transfer function with respect to $\hat{\gamma}_i$ and take its expectation to obtain an equation of the same form as (20):

$$E\left(\frac{\partial t_i}{\partial \hat{\gamma}_i}(\hat{\gamma}_i = \gamma_i)\right) = \alpha_{1i} v_i s_i - \alpha_{2i} \left(\sum_{j \neq i} w_j \right) v_i s_i - 2\alpha_{2i} w_i (s_i + \theta_i / v_i) \quad (21)$$

This equation can be rearranged slightly to facilitate matching coefficients with equation (20):

Matching the θ_i coefficients immediately reveals:

$$\alpha_{2i} = c \left(\frac{v_i}{w_i} - 1 \right) \quad (22)$$

Matching the s_i coefficients in equation (21) simplifies to:

$$\alpha_{1i} = c(1 + \frac{v_i}{w_i}) \left(\sum_{j \neq i} w_j \right) \quad (23)$$

Because $v_i > w_i$ as long as $V_\theta \geq 0$ and $V_i \geq 0$ with at least one inequality being strict, this implies that $\alpha_{1i} > 0$ and $\alpha_{2i} > 0$. This means that the transfer function hypothesized above is concave in $\hat{\gamma}_i$. Combining this concave transfer with the concave decision rule yields a total transfer function that satisfies the second-order condition for incentive compatibility.³⁰

The final step is to calculate the expectation of the above transfer $E(t_i(\gamma, y))$ and set it to zero to solve for \bar{t}_i . The interpretation is that \bar{t}_i is the expected trading profits of expert i excluding any lump sum payments. Note that:

$$\begin{aligned} \bar{t}_i &= E \left(\alpha_{1i} \gamma_i (y - \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \gamma_j) \right) = \\ &\alpha_{1i} Cov(\gamma_i, y) - \alpha_{2i} \sum_{j \neq i} Cov(\gamma_i, \gamma_j) - \alpha_{2i} Var(\gamma_i) = \\ &cw_i \left[2V_0 \sum_{j \neq i} w_j - w_i \left(V_i + \frac{1}{v_i^2} V_\theta \right) \right] \end{aligned} \quad (24)$$

All the parameters in the equation for \bar{t}_i are publicly observable. Thus, $t_i(\hat{\gamma}, y) = \alpha_{1i} \hat{\gamma}_i (y - \frac{\alpha_{2i}}{\alpha_{1i}} \sum_j \hat{\gamma}_j) - \bar{t}_i$ is an optimal transfer function with a zero expectation. QED.

Proposition 3: When V_θ is sufficiently small, then $\frac{\alpha_2}{\alpha_1} < 1$. However, when $V_\theta \geq V_0$, then $\frac{\alpha_2}{\alpha_1} \geq 1$.

Proof: We have $\alpha_{1i} = (2c + \alpha_{2i}) \left(\sum_{j \neq i} w_j \right) = c(\frac{v_i}{w_i} + 1) \left(\sum_{j \neq i} w_j \right)$.

Thus, $\alpha_{1i} - \alpha_{2i} = 2c \left(\sum_{j \neq i} w_j \right) + \alpha_{2i} \left(\sum_{j \neq i} w_j - 1 \right)$. After substituting $\alpha_{2i} = c(\frac{v_i}{w_i} - 1)$, this can be simplified to:

³⁰Experts' individual rationality constraints do not bind because the decision maker can impose arbitrarily large punishments on experts by choosing an undesirable decision. By imposing a decision outcome loss on any non-participating trader equal to or greater than the entrance fee, the decision maker can guarantee that all experts participate. Specifically, if any expert does not participate, the decision maker can make a very high judgment half the time and a very low judgment half the time. This induces all experts to report and accept the decision made based on all experts' reports.

$$\alpha_{1i} - \alpha_{2i} = c \left(\sum_{j \neq i} w_j \right) \left[2 + \left(\frac{v_i}{w_i} - 1 \right) \left(1 - \frac{1}{\sum_{j \neq i} w_j} \right) \right] \quad (25)$$

Recall the definition of $w_i = \frac{(V_i + \frac{1}{v_i^2} V_\theta)^{-1}}{V_0^{-1} + \sum_j (V_j + \frac{1}{v_j^2} V_\theta)^{-1}} = \frac{z_i^{-1}}{V_0^{-1} + \sum_j z_j^{-1}}$, where $z_j = V_j + \frac{1}{v_j^2} V_\theta$ for all j and $v_j = \frac{V_0}{V_0 + V_j}$. We can now evaluate the term inside the square brackets in equation (25):

$$2 + \left(\frac{\frac{V_0}{V_0 + V_j}}{\frac{z_i^{-1}}{V_0^{-1} + \sum_j z_j^{-1}}} - 1 \right) \left(1 - \frac{1}{\sum_{j \neq i} \frac{z_i^{-1}}{V_0^{-1} + \sum_j z_j^{-1}}} \right) \quad (26)$$

After considerable simplification, this becomes:

$$1 - \left(\frac{V_\theta}{V_0} \right) \left(1 + \frac{V_i}{V_0} \right) \left[1 + \frac{V_0 + z_i}{V_0 z_i \left(\sum_{j \neq i} z_j^{-1} \right)} \right] \quad (27)$$

The second term is less than 1 when V_θ is sufficiently small, implying that $\alpha_{1i} - \alpha_{2i} > 0$ or $\frac{\alpha_{2i}}{\alpha_{1i}} < 1$. However, when $V_\theta \geq V_0$, then the all three multiplied terms in the second term are greater than or equal to 1, implying that $\alpha_{1i} - \alpha_{2i} \leq 0$ or $\frac{\alpha_{2i}}{\alpha_{1i}} \geq 1$. QED.

Proposition 4: When V_θ is sufficiently small, then $\frac{\partial \frac{\alpha_{2i}}{\alpha_{1i}}}{\partial V_i} > 0$.

Proof: Substituting the expressions for α_{1i} and α_{2i} from the text, we obtain $\frac{\alpha_{1i}}{\alpha_{2i}} = \left(\sum_{j \neq i} w_j \right) \left(1 + \frac{2c}{\alpha_{2i}} \right) = \left(\sum_{j \neq i} w_j \right) \left(1 + \frac{2w_i}{v_i - w_i} \right)$. To calculate the derivative of this expression, we will first calculate the derivative of various terms.

By definition, recall that $w_j = \frac{(V_j + \frac{1}{v_j^2} V_\theta)^{-1}}{V_0^{-1} + \sum_j (V_j + \frac{1}{v_j^2} V_\theta)^{-1}}$. Differentiating this expression for $j \neq i$, we obtain $\frac{\partial w_j}{\partial V_i} = \frac{w_j (1 + \frac{2(V_i + V_m)}{V_m} V_\theta)}{(V_m^{-1} + \sum_j z_j^{-1}) [V_i + (\frac{V_i + V_m}{V_m})^2 V_\theta]^2} > 0$.

Differentiating w_i with respect to V_i , we find $\frac{\partial w_i}{\partial V_i} = \frac{(1 - w_i)(1 + \frac{2(V_i + V_0)}{V_0} V_\theta)}{(V_0^{-1} + \sum_j z_j^{-1}) [V_i + (\frac{V_i + V_0}{V_0})^2 V_\theta]^2} < 0$.

Next, we compute $\frac{\partial \left(\frac{w_i}{v_i - w_i} \right)}{\partial V_i} = \frac{1}{(v_i - w_i)^2} \left[v_i \frac{\partial w_i}{\partial V_i} - w_i \frac{\partial v_i}{\partial V_i} \right]$. Also, note that $\frac{\partial v_i}{\partial V_i} = \frac{-V_m}{(V_i + V_m)^2} < 0$.

Using these terms, we can now solve for:

$$\begin{aligned} \frac{\partial \frac{\alpha_{1i}}{\alpha_{2i}}}{\partial V_i} &= \frac{\left(\sum_{j \neq i} w_j\right) \left(1 + \frac{2w_i}{v_i - w_i}\right) \left(1 + \frac{2(V_i + V_0)}{V_m} V_\theta\right)}{\left(V_0^{-1} + \sum_j z_j^{-1}\right) \left[V_i + \left(\frac{V_i + V_0}{V_0}\right)^2 V_\theta\right]^2} \\ &+ \frac{2 \left(\sum_{j \neq i} w_j\right)}{(v_i - w_i)^2} \left[\frac{-(1 - w_i) \left(1 + \frac{2(V_i + V_0)}{V_m} V_\theta\right) v_i}{\left(V_0^{-1} + \sum_j z_j^{-1}\right) \left[V_i + \left(\frac{V_i + V_0}{V_0}\right)^2 V_\theta\right]^2} + \frac{w_i V_0}{(V_i + V_0)^2} \right] \end{aligned} \quad (28)$$

After substituting $V_\theta = 0$ and simplifying considerably, we find $\frac{\partial \frac{\alpha_{1i}}{\alpha_{2i}}}{\partial V_i} = \frac{-(\sum_{j \neq i} V_j^{-1})}{(V_0^{-1} + \sum_j V_j^{-1}) V_i^2} <$

0. This implies that $\frac{\partial \frac{\alpha_{2i}}{\alpha_{1i}}}{\partial V_i} > 0$ for V_θ sufficiently small. QED.

Proposition 5: A competitive market maker for decision-relevant information will set $\lambda_c = \frac{c\sqrt{V_0}}{2c\sqrt{V_\theta} - \sqrt{V_0}}$ when $V_\theta > \frac{V_0}{4}$ and refuse to operate otherwise. The decision rule is $x = p = s/2 + \theta/2$ when the market maker is competitive. A competitive market maker supplies more liquidity (λ_c^{-1} is higher) when experts have stronger decision objectives (c increases), more uncertain objectives (V_θ increases), or less information (V_0 decreases). Informed traders will follow the strategy $q_I = \frac{2c\sqrt{V_\theta} - \sqrt{V_0}}{2c\sqrt{V_0}} s$ and manipulators will follow the trading strategy $q_M = \frac{2c\sqrt{V_\theta} - \sqrt{V_0}}{2c\sqrt{V_\theta}} \theta$ when $V_\theta > \frac{V_0}{4}$. Otherwise, both types will not trade.

Proof: We must verify that such an equilibrium exists. First, note that when the decision maker cannot credibly commit to a price-contingent rule, then k will be chosen such that $x(p) = E(y|Q)$. This implies that:

$$k = \frac{E(y|Q)}{p} = \frac{Cov(y, Q)}{\lambda Var(Q)} \quad (29)$$

Given her decision rule in (10), the decision maker still must decide how much to subsidize liquidity provision (λ) in the market.

We can substitute the hypothesized pricing and trading rules from (7) and (9) into the informed trader's utility maximization problem:

$$q_I \in \arg \max_{\hat{q}_I} \{E_{y,s,\theta} [\hat{q}_I(y - \lambda(q_M + \hat{q}_I))]\} \quad (30)$$

Differentiating the informed trader's objective function and using the distributional assumptions, we obtain the first-order condition:

$$q_I = \frac{1}{2\lambda} s \quad (31)$$

Matching coefficients in the derived and assumed informed trading rules ((31) and (8)), we can solve for b_I :

$$b_I = \frac{1}{2\lambda} \quad (32)$$

Next, we solve the manipulator's quantity maximization problem using analogous techniques to find:

$$b_M = \frac{ck}{1 + ck^2\lambda} \quad (33)$$

$$q_M = \frac{ck}{1 + ck^2\lambda} \theta \quad (34)$$

Now we can compute aggregate order flow from informed traders and uninformed manipulators in terms of the pricing and decision rule parameters λ and k :

$$Q = \frac{1}{2\lambda} s + \left(\frac{ck}{1 + ck^2\lambda} \right) \theta \quad (35)$$

Using this order flow expression, we can calculate the expectation of the decision-relevant information conditional on aggregate order flow:

$$E(y|Q) = \frac{Cov(y, Q)}{Var(Q)} Q = \frac{\frac{1}{2\lambda} V_0 \left[\frac{1}{2\lambda} s + \left(\frac{ck}{1 + ck^2\lambda} \right) \theta \right]}{\frac{1}{4\lambda^2} V_0 + \left(\frac{ck}{1 + ck^2\lambda} \right)^2 V_\theta} \quad (36)$$

Equation (36) gives the equilibrium price that would prevail in the decision market if a competitive market maker were used rather than the decision maker. Thus, we find that $\lambda_c = \frac{\frac{1}{2\lambda_c} V_0}{\frac{1}{4\lambda_c^2} V_0 + \left(\frac{ck}{1 + ck^2\lambda_c} \right)^2 V_\theta}$ is the liquidity parameter that a competitive market maker would set. Simplifying this λ_c equation and substituting the appropriate value for k using equation (29) above, we obtain:

$$\lambda_c = \frac{\sqrt{V_0}}{c(2\sqrt{V_\theta} - \sqrt{V_0})} \quad (37)$$

when $V_\theta > \frac{V_0}{4}$. If $V_\theta \leq \frac{V_0}{4}$, a competitive market maker will refuse to accept orders because the adverse selection problem is too severe for him to break even for any value of λ_c .

We can solve for the decision rule parameter k when the market maker is competitive by plugging equation (37) for λ_c into equation (29) to obtain $k = 1$, which implies $x = p$. By substituting equation (37) for λ_c into equations (32) and (33), we obtain the solutions for the informed trader's strategy and the manipulator's strategy.

Now we have verified the equation for λ_c , the decision rule $x = p$, and the traders' equilibrium strategies. To establish the comparative statics for liquidity, we need to take derivatives of λ_c with respect to c , V_θ , and V_0 :

$$\frac{d\lambda_c}{dc} = \frac{-\sqrt{V_0}}{c^2 (2\sqrt{V_\theta} - \sqrt{V_0})} < 0 \quad (38)$$

$$\frac{d\lambda_c}{dV_\theta} = \frac{-\sqrt{V_0/V_\theta}}{c(2\sqrt{V_\theta} - \sqrt{V_0})^2} < 0 \quad (39)$$

$$\frac{d\lambda_c}{dV_0} = \frac{\sqrt{V_\theta/V_0}}{c(2\sqrt{V_\theta} - \sqrt{V_0})^2} > 0 \quad (40)$$

The derivatives of the reciprocal λ_c^{-1} have the opposite sign. QED.

Proposition 6: When $V_\theta \geq \frac{V_0}{4}(1 + c^{-1})^2$, a decision maker who makes a market for decision-relevant information will select a pricing rule $p = \lambda^*Q$ such that $\lambda^* = \lambda_c$, where λ_c is the corresponding pricing rule for a competitive market maker satisfying $\lambda_c = \frac{\sqrt{V_0}}{c(2\sqrt{V_\theta} - \sqrt{V_0})}$. At these values of V_θ , the decision maker will not intervene in the market. However, for intermediate values in which $\frac{V_0}{4} < V_\theta < \frac{V_0}{4}(1 + c^{-1})^2$, the decision maker will act as a market maker with a price sensitivity of $\lambda^* < \lambda_c$. Furthermore, for $0 < V_\theta \leq \frac{V_0}{4}$, the decision maker will set a finite λ^* , whereas a competitive market maker will not operate. In the cases in which $V_\theta < \frac{V_0}{4}(1 + c^{-1})^2$, the decision market maker incurs trading losses, but obtains more precise decision-relevant information than the competitive market maker.

Proof: Now we consider the case in which the decision maker also makes the market for decision-relevant information. Before solving for the decision maker's liquidity parameter choice λ , we need to solve for the coefficient k that she will use in her decision rule $x = kp$. Using equation (29), substituting expressions for $Var(Q)$ and $Cov(y, Q)$ and simplifying, we obtain an implicit solution for $k(\lambda)$:

$$(k(\lambda) - 2)(1 + ck(\lambda)^2\lambda)^2 + 4\lambda^2c^2k(\lambda)^3\frac{V_\theta}{V_0} = 0 \quad (41)$$

When $V_\theta = \frac{V_0}{4}$, then k approaches 1 as $\lambda \rightarrow \infty$. When V_θ approaches zero, then k approaches

2 as $\lambda \rightarrow \infty$. Also, note that we can obtain an implicit expression for $\frac{dk}{d\lambda}$ by implicitly differentiating equation (41):

$$\frac{dk}{d\lambda} = -\frac{(k-2)2ck^2(1+ck^2\lambda) + 8\lambda c^2 k^3 \frac{V_\theta}{V_0}}{(1+ck^2\lambda)^2 + 4ck\lambda(1+ck^2\lambda)(k-2) + 12\lambda^2 c^2 k^2 \frac{V_\theta}{V_0}} \quad (42)$$

The decision maker will anticipate that she will use a decision rule $k(\lambda)$ when she picks her liquidity parameter λ to maximize her utility. Her utility depends on both the conditional variance of decision-relevant information and her trading profits—*i.e.*, her liquidity subsidy costs. Using the decision maker's pricing rule (7) and the aggregate order flow equation (35), we obtain an expression for her expected trading profits:

$$E[Q(p-y)] = \lambda V - C \text{ subject to } \lambda V - C \leq 0 \quad (43)$$

where $C = Cov(y, Q) = V_0/2\lambda$ and $V = Var(Q) = (1/2\lambda)^2 V_0 + \left(\frac{ck(\lambda)}{1+\lambda ck(\lambda)^2}\right)^2 V_\theta$. Recall that the decision maker cannot earn positive profits as a market maker because she faces perfect competition from other uninformed market makers. Note that the decision maker accepts a trading loss if she sets $\lambda < C/V = \lambda_c$.

We can compute the informativeness of order flow using the expressions for aggregate order flow Q in equation (35) and the conditional expectation of decision-relevant information $E(y|Q)$ in equation (36). Substituting these expressions in the definition of conditional variance $Var(y|Q) = E[(y - E(y|Q))^2]$, we obtain:

$$Var(y|Q) = \frac{\left(\frac{ck(\lambda)}{1+\lambda ck(\lambda)^2}\right)^2 V_\theta V_0}{(1/2\lambda)^2 V_0 + \left(\frac{ck(\lambda)}{1+\lambda ck(\lambda)^2}\right)^2 V_\theta} = V_0 - \frac{C^2}{V} \quad (44)$$

Combining the expressions for the decision maker's trading profits (43) and the noise in order flow information (44), we can write the decision maker's problem as:

$$\max_{\lambda} \left[\lambda V - C - V_0 + \frac{C^2}{V} \right] \quad (45)$$

subject to the constraints that λ and k satisfy equation (41) and $\lambda V - C < 0$.

We can now simplify the decision maker's utility function by substituting equation (29) to obtain an unconstrained maximization problem in terms of λ and $k(\lambda)$:

$$\max_{\lambda} \left[\frac{1 - k(\lambda)}{\lambda k(\lambda)} + k(\lambda) \right] \text{ subject to } k(\lambda) \geq 1 \quad (46)$$

where we have implicitly imposed the constraint by allowing k to depend on λ through equation (41). Also, we have imposed $k(\lambda) \geq 1$, which is equivalent to the competition constraint that prevents the decision maker from obtaining positive expected trading profits. This equivalence holds because non-positive expected profits means $\lambda V - C \leq 0$, which implies $k = \frac{C}{\lambda V} \geq 1$ from equation (29).

We solve the maximization problem in (46) by evaluating several cases. First, suppose that $\frac{V_{\theta}}{V_0} > \frac{1}{4}$, which means that a competitive market maker would supply some finite level of liquidity to the market. To assess how much liquidity the decision maker would supply, we can evaluate the derivative of the above expression under the alternative assumptions that $k = 1$ and $k > 1$.

Now let's investigate whether $k = 1$ is an equilibrium. Again, using equation (29), we can infer that $\lambda = \lambda_c$. To assess whether this choice of λ is optimal for the decision maker, we must calculate her marginal utility of λ at λ_c and $k = 1$. We will focus on the marginal utility of λ as λ approaches λ_c from below. This marginal utility must be non-negative at $\lambda = \lambda_c$ and $k = 1$ for this to be an equilibrium.³¹ The relevant marginal utility is:

$$\frac{dU}{d\lambda} = \frac{k-1}{\lambda^2 k} - \frac{dk}{d\lambda} \frac{1}{\lambda k^2} + \frac{dk}{d\lambda} \text{ when } \lambda < \lambda_c \quad (47)$$

Evaluating this marginal utility at $k = 1$ and $\lambda = \lambda_c = \frac{\sqrt{V_0}}{c(2\sqrt{V_{\theta}} - \sqrt{V_0})}$, we obtain:

$$\frac{dU}{d\lambda} = \frac{1}{2\lambda_c} \left(\frac{1}{\lambda_c} - 1 \right) \geq 0 \Leftrightarrow \lambda_c \leq 1 \Leftrightarrow V_{\theta} \geq \frac{V_0}{4}(1 + c^{-1})^2 \quad (48)$$

which is consistent with our initial assumption that $\frac{V_{\theta}}{V_0} > \frac{1}{4}$. We conclude that $\lambda^* = \lambda_c$ and $k = 1$ is optimal for the decision maker when the market is sufficiently liquid under competition (*i.e.*, $\lambda_c \leq 1$), which occurs when there is a sufficient degree of manipulative trade (*i.e.*, $V_{\theta} \geq \frac{V_0}{4}(1 + c^{-1})^2$).

³¹We have also verified that $dU/d\lambda < 0$ for $\lambda < \lambda_c$, which (along with equation (48)) is sufficient to guarantee that $k = 1$ and $\lambda = \lambda_c$ is an equilibrium.

We can reuse the marginal utility equation (48) to assess the situations in which $k > 1$ and $\lambda^* < \lambda_c$ is an equilibrium. Note that the marginal utility of λ is strictly negative at $\lambda^* = \lambda_c$ and $k = 1$ when $\frac{V_0}{4} < V_\theta < \frac{V_0}{4}(1 + c^{-1})^2$. This implies the decision maker would like to lower λ below the competitive level, which means that $\lambda^* < \lambda_c$ and $k > 1$ is an equilibrium. Furthermore, k must be less than 2 to satisfy equation (41), implying that $1 < k < 2$.

Now we consider the case in which $\frac{V_\theta}{V_0} \leq \frac{1}{4}$. In this case, a competitive market maker will not supply any liquidity to the market because there is no finite λ at which a market maker can make non-negative trading profits. To show that a decision market maker will set a finite value of λ , we only need to show that she receives a greater utility by accepting arbitrarily small trading losses than she receives when the market is closed.

From equation (46), we see that the decision maker's utility is zero when the market is closed because she incurs no trading losses and her decision will not depend on the (non-existent) market price. Now we must compare this utility to her utility from setting a finite, but arbitrarily high, value of λ .

First, we solve for the value of k that is consistent with $\lambda \rightarrow \infty$. Taking the limit of both sides of equation (41) for k , we obtain $\lim_{\lambda \rightarrow \infty} k = 1 \pm \sqrt{1 - 4\frac{V_\theta}{V_0}}$. Only the $k > 1$ solution is relevant because, as noted earlier, $k < 1$ is incompatible with the non-positive trading profits in a competitive equilibrium. Now we can insert this expression for the limit of k in the decision maker's utility function to obtain a limiting utility of $k = 1 + \sqrt{1 - 4\frac{V_\theta}{V_0}} > 0$ when $\lambda \rightarrow \infty$. We conclude that the decision maker is better off keeping the market open and accepting a small trading loss.

Finally, we can infer that the decision maker is obtaining better decision-relevant information—*i.e.*, $\text{Var}(y|p = \lambda^*Q) < \text{Var}(y|p = \lambda_cQ)$ —in the cases in which she is accepting trading losses. This must be true in order for λ^* to be a maximum. QED

Proposition 7: The equilibrium market liquidity (λ^{-1}), decision rule, and manipulator and informed trading strategies are the same whether a rational decision maker optimally trades in the market or optimally makes the market.

Proof: We can establish an equivalence between the two equilibria by showing that the decision maker solves the same constrained optimization problem in both models. In the market making model, we have already shown that the objective function is given by (45). The decision maker maximizes this function subject to the constraint given by equation (41). Thus, the proof consists of writing down the analogous problem for a decision maker who opts to randomly trade in the market with a competitive market maker.

In the random trading model, the decision maker submits a random order ρ , where $\rho \sim N(0, V_\rho)$, to the competitive market maker. The decision maker selects the distribution of the order, but cannot control the specific realization of her order. Because the single parameter V_ρ is a sufficient statistic for the distribution of ρ , we can think of the decision maker as choosing V_ρ . For simplicity, we suppose that only the decision maker can observe the realization of ρ , which occurs after the order is submitted but before the market clears. This will allow the decision maker to “debias” the market price by accounting for the effect of the realization of ρ . The competitive market maker only observes aggregate order flow, but is aware that the decision maker’s random order flow is included in aggregate order flow.

To show the market making and random trading problems faced by the decision maker are analogous, we will first show that the constraints are analogous. Then we will show that the objective functions are analogous, implying that the solutions must be the same. Throughout the proof, we will denote the variables and parameters in the random trading model by the same notation as in the market making model except for the subscript T —*e.g.*, the equilibrium price is given by p_T rather than p .

Because he treats the decision maker as a noise trader, the competitive market maker maintains a simple linear pricing rule given by $p_T = \lambda_T(Q_T + \rho)$, where $Q_T = q_{IT} + q_{MT}$. After she observes the realization of ρ , the decision maker can infer the aggregate order flow of all other traders by inverting the equilibrium price:

$$p_T/\lambda_T - \rho = Q_T \quad (49)$$

The aggregate order flow of the informed agent and the manipulator (Q_T) is a sufficient statistic for the decision maker’s inference about the conditional expectation of y . Applying the conditional expectation formula for jointly normally distributed variables, we obtain:

$$E(y|Q_T) = \frac{C_T}{V_T} Q_T \quad (50)$$

where we have defined $C_T = Cov(y, Q_T)$ and $V_T = Var(Q_T)$. Note that these definitions imply $C_T = C$ and $V_T = V$, which will be useful later.

As before, we assume the decision maker cannot commit to any decision other than her ex post optimal choice, which is $x_T = E(y|Q_T)$. Using equations (50) and (49), this choice can be expressed as:

$$x_T = \frac{C_T}{V_T}(p_T/\lambda_T - \rho) = k_T Q_T + k_\rho \rho \quad (51)$$

where we have defined two decision rule parameters, $k_T = \frac{C_T}{\lambda_T V_T}$ and $k_\rho = -\frac{C_T}{V_T}$, to facilitate our analogy with the market making model. Moreover, we note that $k_T = \frac{C_T}{\lambda_T V_T}$ is identical to the expression for k in the market making model (after substituting the random trading model parameters).

Because the decision in the random trading model depends linearly on the equilibrium price just as the decision in the market model did, we can reuse the formulas for the informed trader's and manipulator's strategies—*i.e.*, equations (33) and (32). Making the appropriate substitutions using the parameters in the random trading model, we obtain $q_{IT} = (1/2\lambda_T)s$ and $q_{MT} = \frac{ck_T}{1+ck_T^2\lambda_T}$. Because each of these expressions is identical (after substituting the random trading model parameters) to its market making model counterpart, this implies that Q_T is directly analogous to Q in the market making model. Combining this fact with the fact that k_T is directly analogous to k , we can derive an equation directly analogous to the market making model constraint on k in (41):

$$(k_T(\lambda_T) - 2)(1 + ck_T(\lambda_T)^2\lambda_T)^2 + 4\lambda_T^2 c^2 k_T(\lambda_T)^3 \frac{V_\theta}{V_0} = 0 \quad (52)$$

This proves that the constraint in the random trading model is isomorphic to the constraint in the market making model. Next, we will show that the objective functions are also isomorphic.

As a random trader, the decision maker again receives profits from two sources. First, the decision maker reaps profits from her random trading activity. Second, the decision maker profits from her production decision, which is informed by the equilibrium market price. Her random trading profits are given by:

$$E(\rho(y - p_T)) = -\lambda_T V_\rho \quad (53)$$

Her production profits are given by:

$$-Var(y|Q_T) = V_0 - \frac{C_T^2}{V_T} \quad (54)$$

Furthermore, the competitive market maker will set λ_T so that he will attain zero profits conditional on aggregate order flow ($Q_T + \rho$). Using the standard Kyle arguments described

earlier, we can solve for the liquidity parameter λ_T :

$$\lambda_T = \frac{Cov(y, Q_T + \rho)}{Var(Q_T + \rho)} = \frac{C_T}{V_T + V_\rho} \quad (55)$$

Note that equation (55) implies there is a one-to-one mapping between V_ρ and λ_T , implying that we can treat the decision maker's random trading problem as one where she chooses λ_T . We rearrange equation (55) slightly, solving for the expected trading profits of the decision maker:

$$-\lambda_T V_\rho = \lambda_T V_T - C_T \quad (56)$$

Using (53), (54) and (56), we can write the decision maker's total utility with random trading as:

$$\lambda_T V_T - C_T + V_0 - \frac{C_T^2}{V_T} \quad (57)$$

which depends only on λ_T .

Because $C_T = C$ and $V_T = V$, this random trading objective function above is isomorphic to the objective function for the decision maker when she makes the market (see (45)). Because both the objective functions and constraints are isomorphic, it follows that the random trading and market making problems have the same equilibria. In other words, $\lambda_T = \lambda$ and $k_T = k$, which implies that $V_\rho = (1/2\lambda_T)^2 V_0 - \left(\frac{ck_T}{1+\lambda_T ck_T^2}\right)^2 V_\theta$. Intuitively, the variance in the order flow from the informed trader must be equal to the total variance in the order flow from uninformed traders—*i.e.*, the manipulator and the randomly trading decision maker. QED.

Appendix B

This appendix discusses and motivates the two modifications to the optimal contract from Section 4 that lead us to consider a simpler decision market in Section 5. First, we suppose that decision makers cannot identify individual traders. Second, we suppose that the decision maker cannot commit to a decision rule that is not in her best interest after the market clears.

In many applications, assigning different contracts to different traders (*e.g.*, $t_i(\hat{\gamma}, y) \neq t_j(\hat{\gamma}, y)$) based on their characteristics will not be feasible. Discrimination may be impossible because certain characteristics are unobservable, illegal because certain characteristics are explicitly protected by law, or unacceptable because certain characteristics are implicitly protected by social norms. But perhaps most importantly, discrimination will create contract resale opportunities for other traders unless the following no-arbitrage conditions are met:

$$t_i(\hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}) + t_j(\hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}) = t_{i+j}(\hat{\gamma}_i + \hat{\gamma}_j, \gamma_{-i,j}), \quad \forall \hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j} \quad (58)$$

$$t_i(\hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}) = t_j(\hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}), \quad \forall \hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j} \quad (59)$$

The first constraint can be viewed as the combination of order flow consolidation and order flow splitting arbitrage restrictions. Specifically, we rule out transfer schemes in which traders can profit by either bundling or splitting their orders.³² The second constraint ensures that transfer rules do not discriminate based on traders' identities.

To see how these constraints can be violated, recall that the optimal contract in section 4 requires that the decision maker impose an entrance fee on each trader participating in the mechanism. This violates no-arbitrage because each trader, after incurring the entrance fee, has an incentive to become a broker for other traders. He can collect other traders' entrance fees and costlessly route these orders to the decision maker by consolidating them with his own order. Sufficient competition from dual-class traders diverting order flow would drive the entrance fee towards zero. Dual-class traders would reap the entire benefits from the optimal contract's liquidity subsidy, forcing the decision maker to incur the entire cost of the subsidy. Thus, feasible contracts must not impose an entrance fee.

The general consequence of no-arbitrage is that the transfer rule must be of the form: $t(\hat{\gamma}, y) = \hat{\gamma}_i f(y, \sum_j \hat{\gamma}_j)$. Note that $t(\hat{\gamma}_i = 0) = 0$, implying there is no entrance fee for a trader that reports $\hat{\gamma}_i = 0$. To simplify the optimal contract, we assume that all traders have

³²Order flow splitting imposes one inequality constraint, while order flow consolidation imposes the other inequality constraint, leading to an equality constraint.

equally precise information and equally strong decision objectives. This modeling restriction allows contracts that satisfy no-arbitrage to satisfy simultaneously all traders' incentive compatibility constraints—*e.g.*, see equation (20) in section 4.

Importantly, any contract that satisfies no-arbitrage and implements a decision rule based on the posterior mean of y ($x = x\left(\sum_j \gamma_j\right)$) will also satisfy the *interim* individual rationality constraint—*i.e.*, all experts will be willing to participate even after observing their private signals and objectives. To see this, note that incentive compatibility implies that a participating expert prefers to report the truth to reporting any other value of $\hat{\gamma}_i$. In particular, he prefers reporting the truth to reporting $\hat{\gamma}_i = 0$, which would provide him with the same transfer ($t_i(\hat{\gamma}_i = 0) = 0$ by no arbitrage) and decision ($x = x\left(\sum_{j \neq i} \hat{\gamma}_j\right)$) as not participating at all. Formally, if the incentive compatibility constraint holds and the decision rule depends only on the sum of reports, then the interim individual rationality constraint must also hold.

There are at least two ways to justify decision rules that depend only on the posterior mean of y . First, the posterior mean ($\sum_j \gamma_j$) is a sufficient statistic for the dependence of the principal's utility on the decision outcome. Although each agent has individual decision objectives and information about the decision, the decision maker cannot allow the decision to have a different dependence on each agent's signal and objectives unless she can identify particular agents. Thus, applying the sufficient statistic theorem in Holmstrom (1979), there is no loss of generality in considering only the class of contracts with decisions that depend only on $(\sum_j \gamma_j)$.³³ Second, if the decision maker cannot commit to a sub-optimal decision, then her decision rule must be the optimal ex post rule, which is given by $x^* = E(y|\eta) = \sum \gamma_i$. Again, this no-commitment contract depends only on the sufficient statistic for the posterior mean.

Using the results from the previous section, we know that the no-commitment decision rule implies the only incentive compatible transfers must include a term of the form $t(\hat{\gamma}, y) = \alpha_1 \hat{\gamma}_i (y - \frac{\alpha_2}{\alpha_1} \sum_j \hat{\gamma}_j)$. Fortunately, this transfer function also satisfies the no-arbitrage requirement above. But, as noted earlier, the entrance fee \bar{t} does not satisfy the no-arbitrage restriction. Thus, when commitment is not possible, we conclude that the optimal contract remains the same as before, except that there is no entrance fee.

To summarize this discussion, we assume traders have equally precise signals and equally

³³There may be other natural justifications for decision rules of this form. For example, we could impose the restriction that decisions cannot be manipulated by coalitions of experts—*e.g.*, $x(\hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}) = x(\hat{\gamma}_i + \hat{\gamma}_j, \gamma_{-i,j})$, $\forall \hat{\gamma}_i, \hat{\gamma}_j, \gamma_{-i,j}$.

strong decision objectives and that the decision maker cannot commit to a suboptimal decision rule. In addition, we rule out contracts that admit arbitrage opportunities. These assumptions allow us to characterize the optimal contract as $t(\hat{\gamma}, y) = \alpha_1 \hat{\gamma}_i (y - \frac{\alpha_2}{\alpha_1} \sum_j \hat{\gamma}_j)$ and $x = \sum_j \hat{\gamma}_j$. This contract is equivalent to an anonymous market for contingent claims on y that are priced at $\frac{\alpha_2}{\alpha_1} \sum_j \hat{\gamma}_j$. The two parameters in the transfer function govern the liquidity subsidy: $\frac{\alpha_2}{\alpha_1}$ determines the sensitivity of price to order flow and α_1 determines the overall size of the subsidy. The decision maker selects a judgement based on the price according to $x = \frac{\alpha_1}{\alpha_2} \left(\frac{\alpha_2}{\alpha_1} \sum_j \hat{\gamma}_j \right)$. For the remainder of the paper, we focus on simple decision markets for contingent contracts of the sort described above.

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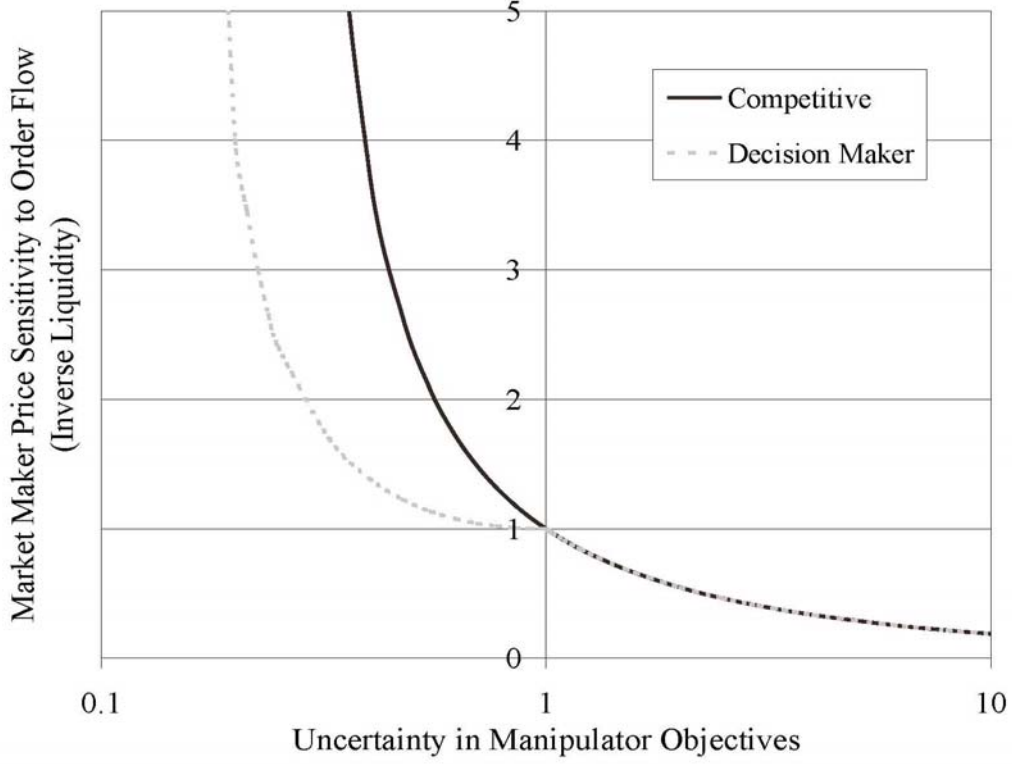


Figure 1. Price sensitivity vs. uncertainty in manipulator objectives. The graph depicts the competitive market maker's price sensitivity to aggregate order flow, λ_c , and the decision market maker's sensitivity, λ^* . Figure 1 shows plots of λ^* and λ_c versus V_θ with the value of $c = 1.0$. We use the analytical solution for λ_c in equation (37) and a numerical solution for λ^* . For the numerical solutions, we impose $V_0 = 1$ and allow V_θ to vary because our results depend on the ratio V_θ/V_0 of the uncertainty in the manipulator's objectives to the prior uncertainty in decision-relevant information.

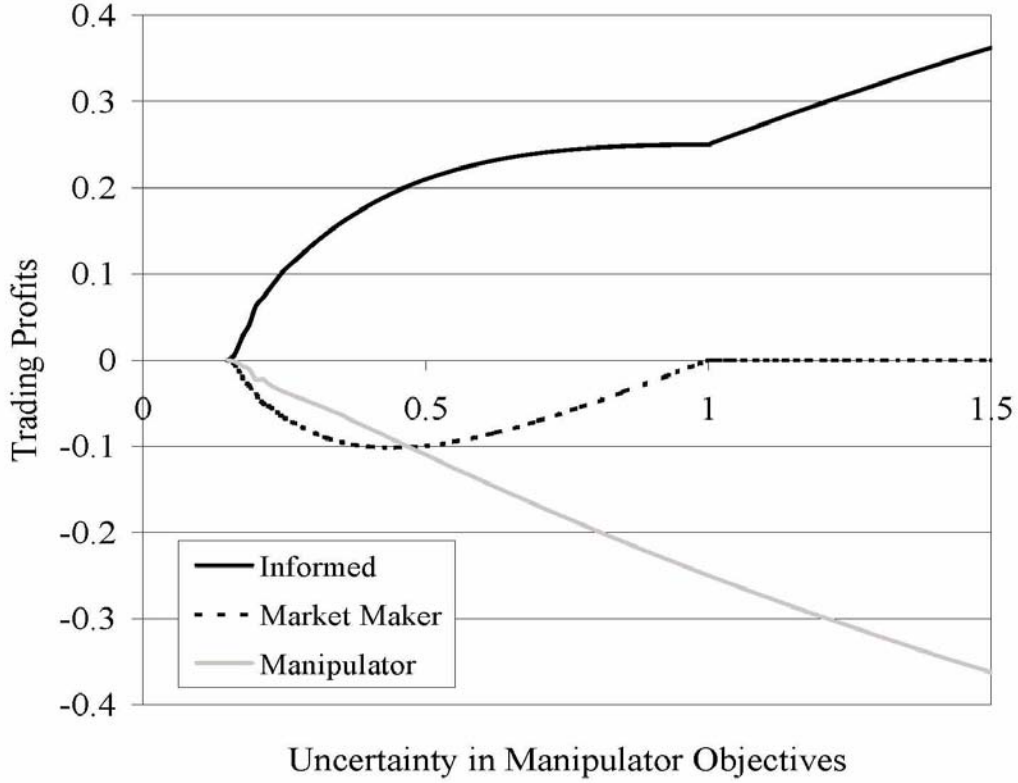


Figure 2. Expected trading profits vs. uncertainty in manipulator objectives.

The graph depicts the expected trading profits of an informed trader, a price manipulator and a decision market maker for the model in section 5. The figure shows these three trading profits as a function of the variance in the price manipulator's objectives over the decision outcome (V_θ). We use the analytical solution for λ_c in equation (37) and a numerical solution for λ^* . For the numerical solutions, we impose $V_0 = 1$ and allow V_θ to vary because our results depend on the ratio V_θ/V_0 of the manipulator's preference uncertainty to the insider's private information. In this figure, we also impose $c = 1$ for simplicity—*i.e.*, the strength of the manipulator's objectives is equal to the strength of the decision maker's.